Induced phase modulation of chirped continuum pulses studied with a femtosecond frequency-domain interferometer

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The effect of rapid phase change on chirped continuum pulses is studied with a frequency-domain interferometer. Because of the chirp, temporal evolution of the optical Kerr response in CS$_2$ is projected into difference phase spectra. The chirped continuum shows spectral shifts that are due to induced phase modulation even when the continuum has a flat spectrum.

When a femtosecond white-light continuum pulse is used in femtosecond spectroscopy, the frequency chirp, which is caused by positive group-velocity dispersion of optical elements, must be phase compensated to obtain true transient spectra, otherwise the spectra will have a wavelength-dependent time-delay distortion. Because of higher-order phase distortion, however, it is difficult to eliminate the relative phase difference completely over the whole spectral range of the continuum even with a combination of negative group-velocity dispersion optical configurations. Instead, delay time correction in the transient spectra is often performed. In this case, however, it is not obvious if it gives us equivalent results to those observed with a chirp-compensated pulse.

In this Letter the effect of rapid phase change on a chirped continuum is studied to show that there is a residual effect of the chirp that cannot be eliminated even after delay time correction. For this purpose, the phase change of the continuum induced by the optical Kerr response of a transparent Kerr liquid CS$_2$ is measured with a frequency-domain interferometer.

The laser source, amplification, and measurement systems were described in a previous Letter. The amplified pulses of 60-fs duration, 620-nm wavelength, 10-kHz repetition rate, and 2-$\mu$J energy are divided into the pump and probe pulses. The probe is focused into an ethylene glycol jet to generate a white-light continuum. The continuum pulse is further divided into the two arms of a Michelson interferometer for the reference and probe, and the two pulses are displaced temporally by a few hundreds of femtoseconds. Then they travel along a common path and are detected by a spectrometer with a multichannel photodiode array after being transmitted through a sample. The pump is blocked by a mechanical shutter at 10 Hz to obtain difference transmission spectra (DTS), which were taken by blocking the reference beam. Figure 1(b) shows normalized interference spectra with and without excitation and the difference phase spectra (DPS; open circles). The DPS were derived by the same procedure as before.

Figure 2 displays the DTS and DPS without delay time correction at three time delays, where zero delay is defined at the maximum overlap between the pump and probe at 620 nm. Since the continuum is positively chirped after passing through optical elements of ~11-mm total thickness, the red region of the probe arrives earlier at the sample than does...
of the probe-field envelope, inverse Fourier transform

\[ Eo(co) = Eo(\omega - \omega_0)\exp[-i\omega(\omega - \omega_0)^2], \quad (1) \]

where \( p \) is a linear chirp parameter. If \( Eo(\omega) = Eo\pi^{1/2} \exp(-\tau_p^2/4) \), where \( \tau_p \) is the 1/e half-width of the probe-field envelope, inverse Fourier transform of Eq. (1) leads to the temporal dependence of the pulse field:

\[ E(t) = F^{-1}[E(\omega - \omega_0)] \]

\[ = Eo(\epsilon - i\gamma)^{1/2} \exp[i\omega_0 - (\epsilon - i\gamma)t^2], \]

where \( \epsilon = \tau_p^2/(\tau_p^4 + 16\rho^2) \) and \( \gamma = 4\rho/(\tau_p^4 + 16\rho^2) \). If \( \tau_p^2 \ll 4\rho, \quad \gamma \sim 1/(4\rho) \). Since \( \Phi(t) = \omega_0 + \gamma t^2 \), the instantaneous frequency is \( \omega = d\Phi/dt = \omega_0 + 2\gamma t \). Therefore, \( \rho \) is obtained from the slope of a \( t-v \) curve as \( \rho \sim (1/2)d\Delta t/d\omega = -(1/2)d\tau/d\omega \). From the fitting function, \( \rho = 350 \text{ fs}^2 \) at 700 nm.

The simulation is performed as follows:

\[ F[E(t)] = R(\omega)\exp[i\Phi(\omega)], \]

\[ F[R(t)\exp[i\Delta \Phi(t - \tau)]] = R(\omega, \tau)\exp[i\Phi(\omega, \tau)], \]

\[ \Delta T/T(\omega, \tau) = \frac{[R^2(\omega, \tau) - R^2(\omega)]/R^2(\omega)}{\Delta \Phi(\omega, \tau) = \Phi(\omega, \tau) - \Phi(\omega)}, \]

where \( E(t) \) is the inverse Fourier transform of the probe-pulse field [Eq. (1)] with \( \rho = 350 \text{ fs}^2 \) and a hyperbolic-secant envelope \( Eo(\omega) \). According to the probe spectra in Fig. 1(a), \( |E(t)|^2 \) is assumed to have a 6-fs FWHM in the Fourier-transform limit, \( \rho = 0 \). \( \Delta \Phi(\omega) \) is a phase change that consists of the electronic and nuclear response terms, whereas the former is assumed to be the same as the pump-pulse intensity function with 60-fs FWHM and the latter is assumed to be the same function as Eq. (3) in Ref. 4. The results are shown in Fig. 4, which reproduces the observed spectral shifts fairly well. It is therefore proved that the DTS signals are caused by induced phase modulation; rise and decay in the DPS with decreasing wavelength, i.e., with increasing time delay, are accompanied by red and blue shifts of the probe frequency, respectively.

The DTS signals can also be explained as follows. Suppose the probe pulse experiences a small phase change \( \Delta \Phi(t) \), which follows a Gaussian pulse-intensity profile with a 1/e half-width of \( \tau_r \) such that

\[ \Delta \Phi(t) = \delta \exp(-t^2/\tau_r^2), \quad \delta \ll 1, \]

\[ E(t)\exp[i\Delta \Phi(t)] \sim E(t)[1 + i\delta \exp(-t^2/\tau_r^2)]. \quad (2) \]
The Fourier transform of the right-hand side of relation (2) is

\[ F[E(t) + iE(t)\Delta\Phi(t)] = E(\omega) + i\delta E_{ex}(\omega), \]

\[ E_{ex}(\omega) = \frac{E_0\pi^{1/2}(\epsilon - i\gamma)^{1/2}}{(1/\tau_e^2 + \epsilon - i\gamma)^{1/2}} \exp\left[ -\frac{-\omega^2}{4(1/\tau_e^2 + \epsilon - i\gamma)} \right]. \]

The DTS is expressed as

\[ \Delta T/T(\omega) = \frac{[E(\omega) + i\delta E_{ex}(\omega)]/E(\omega)]^2 - 1}{-2\delta \text{ Im}[E_{ex}(\omega)/E(\omega)]} \]

\[ = -2\delta \text{ Im}((\epsilon - i\gamma)^{1/2}/(1/\tau_e^2 + \epsilon - i\gamma)^{1/2}) \times \exp[-\omega^2/[4(1/\tau_e^2 + \epsilon - i\gamma)] + \tau_p^2\omega^2/4 + i\rho\omega^2]. \]

In the limit of a broad probe spectrum (\( \tau_p \to 0 \)),

\[ \Delta T/T(\omega) \sim -2\delta \text{ Im}((-i\gamma)^{1/2}/(1/\tau_e^2 - i\gamma)^{1/2}) \times \exp[-\omega^2/[4(1/\tau_e^2 - i\gamma)] + i\rho\omega^2]. \]

Further, when \( \rho \) is much larger than \( \tau_e^2 \) [\( \gamma = 1/(4\rho) \ll 1/\tau_e^2 \)] and \( \rho > 0 \),

\[ \Delta T/T \sim \delta(1/2\rho)^{1/2}\tau_e \exp(-\tau_e^2\omega^2/4) \times (\cos\rho\omega^2 - \sin\rho\omega^2). \]

The oscillation observed in the DTS can be explained qualitatively by relation (4), although the condition \( \gamma \ll 1/\tau_e^2 \) is not satisfied in the present experiment because \( \rho = 350 \text{ fs}^2 \) and \( \tau_e = 43 \text{ fs} \).

It should be noted that frequency shifts due to induced phase modulation are observed even when the probe spectrum is flat; the shifts depend not only on the spectral shape but also on the chirp. When the continuum has a flat spectrum (\( \tau_p = 0 \)) and is not chirped (\( \rho = 0 \)), no signal is observed in DTS (\( \Delta T/T = 0 \)), as readily derived from relation (3). This is because all the frequency components of the probe have the largest amplitude at the same time such that the frequency shift of any component is canceled by the shift of its neighboring components. When the continuum is chirped, on the other hand, the frequency and time are related linearly through \( \Delta \omega = 2\gamma\Delta t \) such that the frequency shift of one component interferes with its neighboring components to give frequency-domain interference as in relation (4).

Even after delay time correction is performed for the signals in Fig. 2, the spectral shifts observed in the DTS cannot be eliminated, so that the corrected spectra will not show the intrinsic dynamics. The DTS and DPS free from the probe-pulse modulation effect can be obtained only with a probe pulse with a flat spectrum and without chirp, i.e., a \( \delta \)-function pulse in the time domain. In femtosecond spectroscopy, therefore, it is essential to use a continuum with as small chirp as possible. In the real measurements, however, the chirp cannot be eliminated completely, so that the original chirp of a continuum should always be taken into account when interpreting the corrected spectra.

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References

1. Z. Bor and B. Racz, Appl. Opt. 24, 3440 (1985), and references therein.


