4-2-b 1202483 渡辺謙仁

$W(E)$は、重複を許しながらN個の準位にM個の量子を配る場合の数に等しいので、

$$W(E)=\frac{(N-1+M)!}{(N-1)!M!}\cong\frac{(N+M)!}{N!M!}$$

Stirlingの公式を用いてエントロピーσを求めると、

$$\sigma=\ln W(E)=\ln(N+M)!-\ln N!-\ln M!$$
$$\cong\{(N+M)\ln(N+M)-(N+M)\}$$
$$-\{N\ln N-N\}-\{M\ln M-M\}$$
$$=N\left\{\left(1+\frac{M}{N}\right)\ln\left(1+\frac{M}{N}\right)-\frac{M}{N}\ln\frac{M}{N}\right\}$$
$$=N\left\{\begin{array}{l}\left(\frac{E}{N\hbar\omega}+\frac{1}{2}\right)\ln\left(\frac{E}{N\hbar\omega}+\frac{1}{2}\right)\\-\left(\frac{E}{N\hbar\omega}-\frac{1}{2}\right)\ln\left(\frac{E}{N\hbar\omega}-\frac{1}{2}\right)\end{array}\right\}$$

$\tilde{\sigma} \equiv \dfrac{\sigma}{N}, \quad \varepsilon \equiv \dfrac{E}{N\hbar\omega}$ **とおくと、**

$$\tilde{\sigma} = \left(\varepsilon + \frac{1}{2}\right)\ln\left(\varepsilon + \frac{1}{2}\right) - \left(\varepsilon - \frac{1}{2}\right)\ln\left(\varepsilon - \frac{1}{2}\right)$$

(i) $\dfrac{E}{N} >> \dfrac{\hbar\omega}{2} \quad \therefore \varepsilon = \dfrac{E}{N\hbar\omega} >> \dfrac{1}{2}$ **のとき**

$$\left(\varepsilon \pm \frac{1}{2}\right)\ln\left(\varepsilon \pm \frac{1}{2}\right) = \left(\varepsilon \pm \frac{1}{2}\right)\ln\varepsilon\left(1 \pm \frac{1}{2\varepsilon}\right)$$

$$= \left(\varepsilon \pm \frac{1}{2}\right)\left\{\ln\varepsilon + \ln\left(1 \pm \frac{1}{2\varepsilon}\right)\right\}$$

$$\cong \left(\varepsilon \pm \frac{1}{2}\right)\left(\ln\varepsilon \pm \frac{1}{2\varepsilon}\right)$$

$$= \varepsilon\ln\varepsilon \pm \frac{1}{2}\ln\varepsilon \pm \frac{1}{2} + \frac{1}{4\varepsilon}$$

$$\therefore \tilde{\sigma} \cong \ln\varepsilon + 1 = \tilde{\sigma}_c$$

(ii) $\dfrac{E}{N} \to \dfrac{\hbar\omega}{2} \quad \therefore \varepsilon = \dfrac{E}{N\hbar\omega} \to \dfrac{1}{2}$ **のとき**

$$\tilde{\sigma} \to 0$$