問4.4(a)

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$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$r = \left(x^2 + y^2 + z^2 \right)^{\frac{1}{2}}$$

$$\frac{\partial}{\partial x} r^{-1} = \frac{\partial r}{\partial x} \frac{\partial}{\partial r} r^{-1} \quad \left(\because chain \quad rule \right)$$

$$= \left\{ \frac{1}{2} \left(x^2 + y^2 + z^2 \right)^{-\frac{1}{2}} \cdot 2x \right\} \cdot \left\{ -r^{-2} \right\}$$

$$= \left\{ x r^{-1} \right\} \cdot \left\{ -r^{-2} \right\}$$

$$= -x r^{-3}$$

$$\therefore \frac{\partial^2}{\partial x^2} r^{-1} = \frac{\partial}{\partial x}\left(-xr^{-3}\right)$$

$$= -r^{-3} - x\frac{\partial}{\partial x} r^{-3} \quad \left(\because \text{積の微分}\right)$$

$$= -r^{-3} + 3x^2 r^{-5}$$

$$\therefore \nabla^2 r^{-1} = \left(-r^{-3} + 3x^2 r^{-5}\right) + \left(-r^{-3} + 3y^2 r^{-5}\right) + \left(-r^{-3} + 3z^2 r^{-5}\right)$$

$$= -3r^{-3} + 3r^2 r^{-5}$$

$$= 0 \quad \left(QED\right)$$