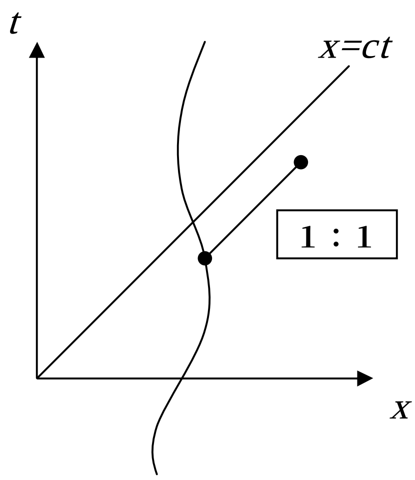
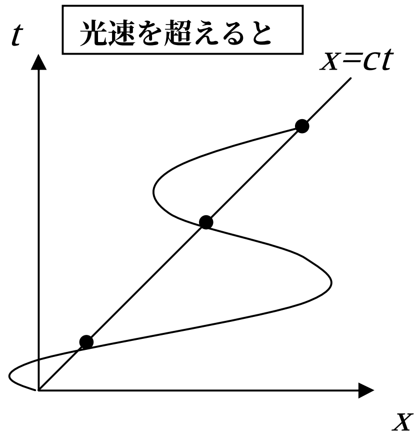
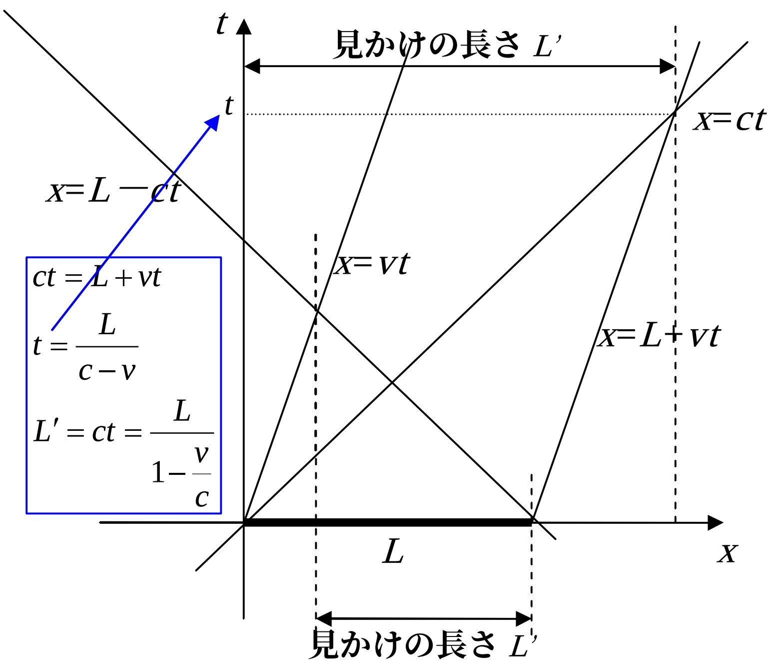
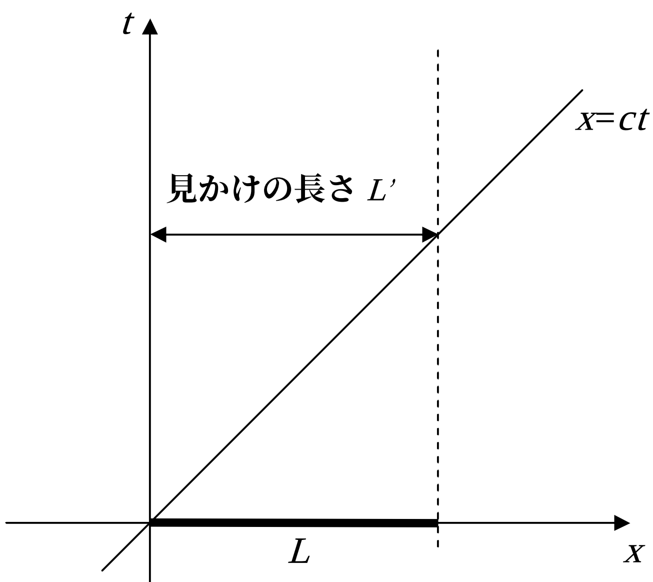
$$V(\mathbf{r},t)=\frac{1}{4\pi\varepsilon_0}\int\frac{q\delta(\mathbf{r}'-\mathbf{w}(t_r))}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}' \quad (10.35)$$

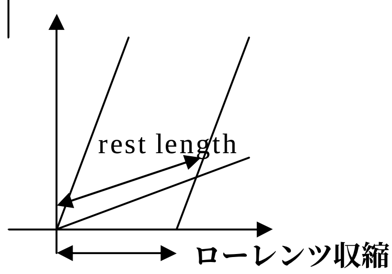
リエナール・ヴィーヒェルトポテンシャルの幾何学的因子　p.431



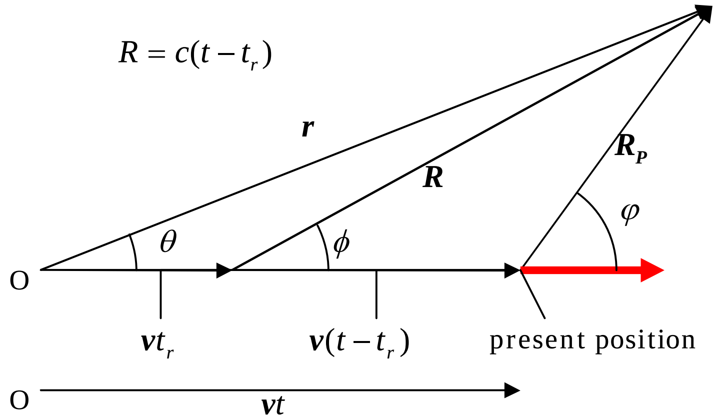








Ex. 10.3 一定速度の点電荷のつくるポテンシャル p.433

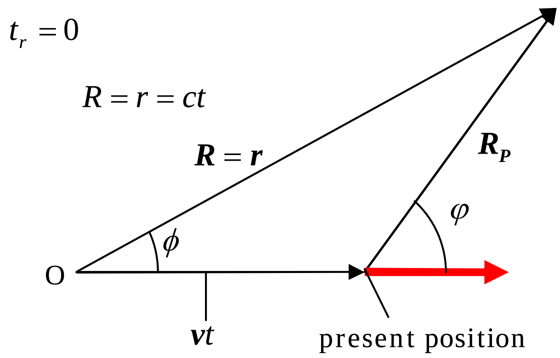


$t_r=0$**のとき** (10.41)**より**

$$c^2t-\boldsymbol{r}\cdot\boldsymbol{v}=\sqrt{(c^2t-\boldsymbol{r}\cdot\boldsymbol{v})^2+(c^2-v^2)(r^2-c^2t^2)}$$

$$V(\boldsymbol{r},t)=\frac{1}{4\pi\varepsilon_0}\frac{qc}{c^2t-\boldsymbol{r}\cdot\boldsymbol{v}}=\frac{1}{4\pi\varepsilon_0}\frac{q}{ct-\boldsymbol{r}\cdot\dfrac{\boldsymbol{v}}{c}}$$

$c>v$(**真空中**)**のとき** $ct-(\boldsymbol{r}\cdot\hat{\boldsymbol{v}})\dfrac{v}{c}=r-(\boldsymbol{r}\cdot\hat{\boldsymbol{v}})\dfrac{v}{c}>0$



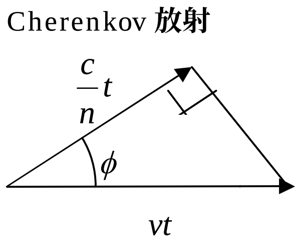
$\dfrac{c}{n}<v$(**媒質中**) $V(\boldsymbol{r},t)$**の分母が**$0\to$ **遠方で減衰しない輻射場**

$$V(\boldsymbol{r},t)=\frac{1}{4\pi\varepsilon_0}\frac{q}{\dfrac{c}{n}t-\boldsymbol{r}\cdot\dfrac{\boldsymbol{v}}{c/n}}$$

$$\frac{c}{n}t-\boldsymbol{r}\cdot\frac{\boldsymbol{v}}{c/n}=0$$

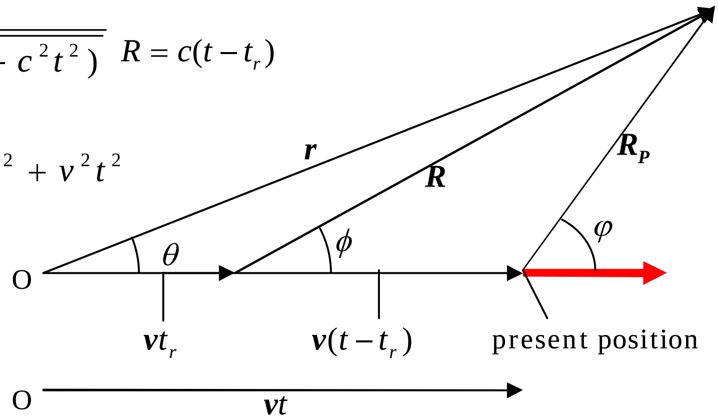
$$(\frac{c}{n})^2t=\boldsymbol{r}\cdot\boldsymbol{v}=rv\cos\phi$$

$r=\dfrac{c}{n}t$**より** $\dfrac{c}{n}=v\cos\phi$



Prob. 10.14

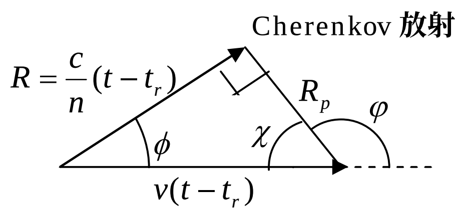
$$V(\boldsymbol{r},t)=\frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{(ct-\boldsymbol{r}\cdot\boldsymbol{v}/c)^2+(1-v^2/c^2)(r^2-c^2t^2)}}$$



$$(ct-\boldsymbol{r}\cdot\boldsymbol{v}/c)^2+(1-v^2/c^2)(r^2-c^2t^2)$$
$$=c^2t^2-2t\boldsymbol{r}\cdot\boldsymbol{v}+(\boldsymbol{r}\cdot\boldsymbol{v})^2/c^2+r^2-c^2t^2-r^2v^2/c^2+v^2t^2$$
$$=-2t\boldsymbol{r}\cdot\boldsymbol{v}+(\boldsymbol{r}\cdot\boldsymbol{v})^2/c^2+r^2-r^2v^2/c^2+v^2t^2$$
$$=r^2-2\boldsymbol{r}\cdot\boldsymbol{v}t+v^2t^2-[r^2v^2/c^2-(\boldsymbol{r}\cdot\boldsymbol{v})^2/c^2]$$
$$=(\boldsymbol{r}-\boldsymbol{v}t)^2-r^2\frac{v^2}{c^2}(1-\cos^2\theta)$$
$$=\boldsymbol{R}_p{}^2-r^2\frac{v^2}{c^2}\sin^2\theta$$
$$=R_p{}^2(1-\frac{v^2}{c^2}\sin^2\varphi)\quad\therefore V(\boldsymbol{r},t)=\frac{1}{4\pi\varepsilon_0}\frac{q}{R_p\sqrt{1-\dfrac{v^2}{c^2}\sin^2\varphi}}$$

$\varphi=\pi-\chi \quad \phi=\pi/2-\chi \quad \sin\varphi=\sin(\pi/2+\phi)=\cos\phi$

$$V(\boldsymbol{r},t)=\frac{1}{4\pi\varepsilon_0}\frac{q}{R_p\sqrt{1-\dfrac{v^2}{(c/n)^2}\sin^2\varphi}}=\frac{1}{4\pi\varepsilon_0}\frac{q}{R_p\sqrt{1-\dfrac{v^2}{(c/n)^2}\cos^2\phi}}$$



運動する点電荷の電磁場　pp.435-438

$\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{w}$

$R = |\boldsymbol{r} - \boldsymbol{w}(t_r)| = c(t - t_r)$

$t_r = t - \dfrac{R}{c}$

$\boldsymbol{w}(t)$: **点電荷の運動の軌跡**

$$\boldsymbol{v} = \left(\frac{d\boldsymbol{w}(t)}{dt}\right)_{t=t_r} = \boldsymbol{v}(t_r)$$

$$V(\boldsymbol{r}, t) = \frac{1}{4\pi\varepsilon_0}\frac{qc}{Rc - \boldsymbol{R}\cdot\boldsymbol{v}}$$

$$\nabla V = \frac{\partial}{\partial \boldsymbol{r}}V(\boldsymbol{r}, t) = \frac{qc}{4\pi\varepsilon_0}\frac{-1}{(Rc - \boldsymbol{R}\cdot\boldsymbol{v})^2}\frac{\partial}{\partial \boldsymbol{r}}(Rc - \boldsymbol{R}\cdot\boldsymbol{v}) \quad (10.49)$$

$$\frac{\partial}{\partial \boldsymbol{r}}R = \frac{\partial t_r}{\partial \boldsymbol{r}}\frac{\partial R}{\partial t_r} = \frac{\partial t_r}{\partial \boldsymbol{r}}(-c) \quad (10.50)$$

$\because R = c(t - t_r(\boldsymbol{r}, t))$　**例えば、**(10.41)

(10.51)**は**p.21**公式**(ii)**より**

$$\boldsymbol{a}\times\nabla t_r = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_x & a_y & a_z \\ \dfrac{\partial t_r}{\partial x} & \dfrac{\partial t_r}{\partial y} & \dfrac{\partial t_r}{\partial z} \end{vmatrix} \quad (10.55)$$

$$\begin{aligned} \nabla(\boldsymbol{R}\cdot\boldsymbol{v}) &= \boldsymbol{a}(\boldsymbol{R}\cdot\nabla t_r) + \boldsymbol{v} - \boldsymbol{v}(\boldsymbol{v}\cdot\nabla t_r) - \boldsymbol{R}\times(\boldsymbol{a}\times\nabla t_r) + \boldsymbol{v}\times(\boldsymbol{v}\times\nabla t_r) \\ &= \boldsymbol{a}(\boldsymbol{R}\cdot\nabla t_r) + \boldsymbol{v} - \boldsymbol{v}(\boldsymbol{v}\cdot\nabla t_r) - \boldsymbol{a}(\boldsymbol{R}\cdot\nabla t_r) + \nabla t_r(\boldsymbol{R}\cdot\boldsymbol{a}) + \boldsymbol{v}(\boldsymbol{v}\cdot\nabla t_r) - \nabla t_r(v^2) \\ &= \boldsymbol{v} + \nabla t_r(\boldsymbol{R}\cdot\boldsymbol{a} - v^2) \quad (10.58) \end{aligned}$$

(10.60)**は**p.21**公式**(ii)**より**

$$\begin{aligned} -c\nabla t_r &= \frac{1}{R}[\boldsymbol{R} - \boldsymbol{v}(\boldsymbol{R}\cdot\nabla t_r) + \boldsymbol{R}\times(\boldsymbol{v}\times\nabla t_r)] \\ &= \frac{1}{R}[\boldsymbol{R} - \boldsymbol{v}(\boldsymbol{R}\cdot\nabla t_r) + \boldsymbol{v}(\boldsymbol{R}\cdot\nabla t_r) - \nabla t_r(\boldsymbol{R}\cdot\boldsymbol{v})] \\ &= \frac{1}{R}[\boldsymbol{R} - \nabla t_r(\boldsymbol{R}\cdot\boldsymbol{v})] \quad \rightarrow (10.61) \end{aligned}$$

$$Rc-\boldsymbol{R}\cdot\boldsymbol{v}=\boldsymbol{R}\cdot\boldsymbol{u}\quad \text{with } \boldsymbol{u}\equiv c\hat{\boldsymbol{R}}-\boldsymbol{v}$$

$$\boldsymbol{E}(\boldsymbol{r},t)=-\nabla V-\frac{\partial\boldsymbol{A}}{\partial t}=-\frac{1}{4\pi\varepsilon_0}\frac{qc}{(Rc-\boldsymbol{R}\cdot\boldsymbol{v})^3}[(Rc-\boldsymbol{R}\cdot\boldsymbol{v})\boldsymbol{v}-(c^2-v^2+\boldsymbol{R}\cdot\boldsymbol{a})\boldsymbol{R}]\quad(10.62)$$

$$-\frac{1}{4\pi\varepsilon_0}\frac{qc}{(Rc-\boldsymbol{R}\cdot\boldsymbol{v})^3}[(Rc-\boldsymbol{R}\cdot\boldsymbol{v})(-\boldsymbol{v}+R\boldsymbol{a}/c)+\frac{R}{c}(c^2-v^2+\boldsymbol{R}\cdot\boldsymbol{a})\boldsymbol{v}]\quad(10.63)$$

$$=-\frac{1}{4\pi\varepsilon_0}\frac{qc}{(\boldsymbol{R}\cdot\boldsymbol{u})^3}\left\{[(\boldsymbol{R}\cdot\boldsymbol{u})\boldsymbol{v}-(c^2-v^2+\boldsymbol{R}\cdot\boldsymbol{a})\boldsymbol{R}]+[(\boldsymbol{R}\cdot\boldsymbol{u})(-\boldsymbol{v}+R\boldsymbol{a}/c)+\frac{R}{c}(c^2-v^2+\boldsymbol{R}\cdot\boldsymbol{a})\boldsymbol{v}]\right\}$$

$$=-\frac{1}{4\pi\varepsilon_0}\frac{qc}{(\boldsymbol{R}\cdot\boldsymbol{u})^3}\left\{(\boldsymbol{R}\cdot\boldsymbol{u})R\boldsymbol{a}/c+(c^2-v^2+\boldsymbol{R}\cdot\boldsymbol{a})(\frac{R}{c}\boldsymbol{v}-\boldsymbol{R})\right\}$$

$$=-\frac{1}{4\pi\varepsilon_0}\frac{q}{(\boldsymbol{R}\cdot\boldsymbol{u})^3}[(\boldsymbol{R}\cdot\boldsymbol{u})R\boldsymbol{a}+(c^2-v^2+\boldsymbol{R}\cdot\boldsymbol{a})(R\boldsymbol{v}-\boldsymbol{R}c)]$$

$$=-\frac{1}{4\pi\varepsilon_0}\frac{q}{(\boldsymbol{R}\cdot\boldsymbol{u})^3}[(\boldsymbol{R}\cdot\boldsymbol{u})R\boldsymbol{a}+(c^2-v^2+\boldsymbol{R}\cdot\boldsymbol{a})(-R\boldsymbol{u})]$$

$$=\frac{q}{4\pi\varepsilon_0}\frac{R}{(\boldsymbol{R}\cdot\boldsymbol{u})^3}[(\boldsymbol{R}\cdot\boldsymbol{a})\boldsymbol{u}-(\boldsymbol{R}\cdot\boldsymbol{u})\boldsymbol{a}+(c^2-v^2)\boldsymbol{u}]\qquad(10.65)$$

Prob. 10.17

Eq.10.33 $\quad c(t-t_r)=R$

$$c^2(t-t_r)^2=R^2=\boldsymbol{R}\cdot\boldsymbol{R}$$

t**で微分** $\quad 2c^2(t-t_r)(1-\frac{\partial t_r}{\partial t})=2\boldsymbol{R}\cdot\frac{\partial\boldsymbol{R}}{\partial t}$

$$cR(1-\frac{\partial t_r}{\partial t})=\boldsymbol{R}\cdot\frac{\partial\boldsymbol{R}}{\partial t}$$

$$\boldsymbol{R}=\boldsymbol{r}-\boldsymbol{w}(t_r)$$

$$\frac{\partial\boldsymbol{R}}{\partial t}=-\frac{\partial\boldsymbol{w}}{\partial t}=-\frac{\partial\boldsymbol{w}}{\partial t_r}\frac{\partial t_r}{\partial t}=-\boldsymbol{v}\frac{\partial t_r}{\partial t}$$

$$cR(1-\frac{\partial t_r}{\partial t})=-\boldsymbol{R}\cdot\boldsymbol{v}\frac{\partial t_r}{\partial t}$$

$$cR=\frac{\partial t_r}{\partial t}(cR-\boldsymbol{R}\cdot\boldsymbol{v})=\frac{\partial t_r}{\partial t}(\boldsymbol{R}\cdot\boldsymbol{u})\quad \text{Eq.10.64}$$

$$\therefore\frac{\partial t_r}{\partial t}=\frac{cR}{\boldsymbol{R}\cdot\boldsymbol{u}}$$