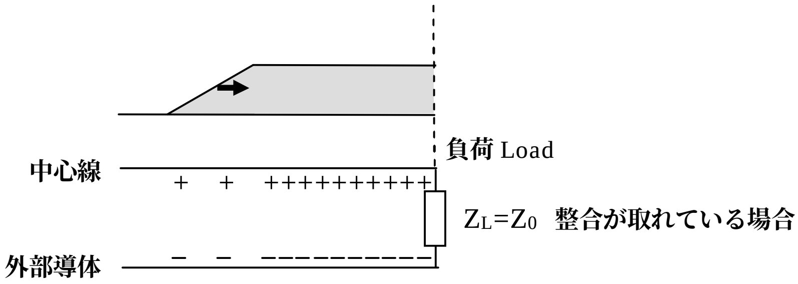
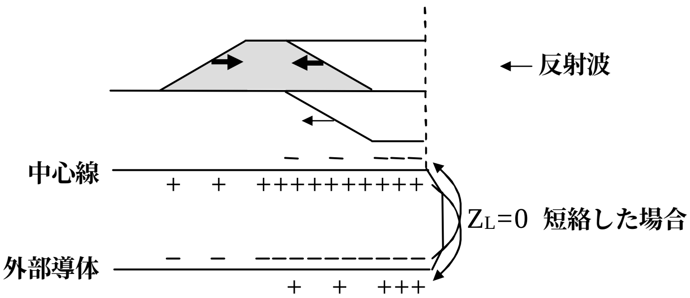
無限に長い同軸ケーブルを伝搬する場合の波形



反射なし

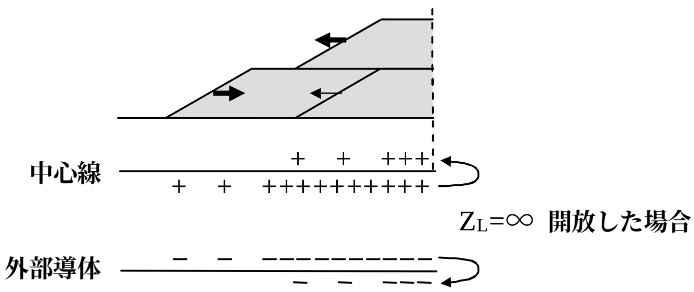
電流と電圧は互いに

見合って打ち消す



符号反転したパルスが反射

電流：無制限
電圧：ゼロ
境界条件



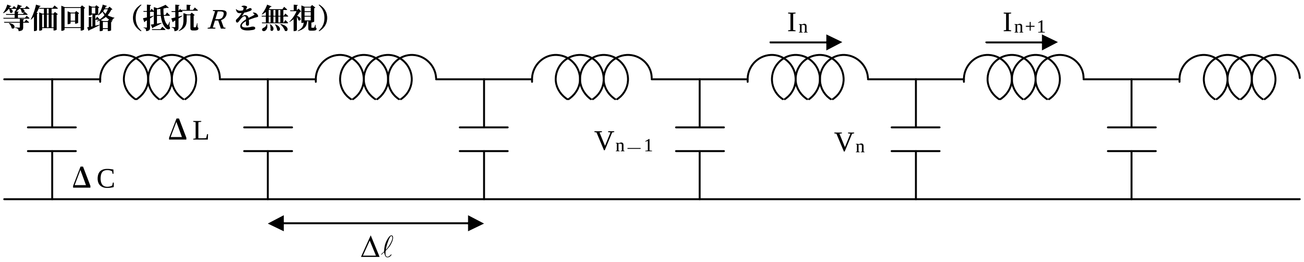
同符号のパルスが反射

電流：ゼロ
電圧：無制限
境界条件

Z_L がキャパシタ、インダクタの場合はどうなるだろうか？

同軸ケーブルでの電磁波の伝搬と反射

等価回路（抵抗 R を無視）



$$Q = CV \quad V = L\frac{dI}{dt} \quad V = V_0 e^{i\omega t} \quad I = I_0 e^{i\omega t}$$

$$V_0 = i\omega L I_0 \quad I = \frac{dQ}{dt} = C\frac{dV}{dt} \quad I_0 = i\omega C V_0$$

$$i\omega \Delta C V_n = I_n - I_{n+1} \quad I_n - I_{n+1} = -\frac{dI}{dx}\Delta\ell$$

$$i\omega \Delta L I_n = V_{n-1} - V_n \quad V_{n-1} - V_n = -\frac{dV}{dx}\Delta\ell$$

$$-i\omega \Delta C V_0 = \frac{dI_0}{dx}\Delta\ell \quad -i\omega \Delta L I_0 = \frac{dV_0}{dx}\Delta\ell$$

$\dfrac{\Delta L}{\Delta\ell} = L' \quad \dfrac{\Delta C}{\Delta\ell} = C'$ **単位長さ当たりのインダクタンス、キャパシタンス**

$$-i\omega C' V_0 = \frac{dI_0}{dx} \quad -i\omega L' I_0 = \frac{dV_0}{dx}$$

$$\frac{d^2 I_0}{dx^2} = -i\omega C'\frac{dV_0}{dx} = -\omega^2 L'C' I_0$$

$$I_0 = Ae^{\pm i\omega\sqrt{L'C'}x} \quad I = Ae^{i\omega(t\pm\sqrt{L'C'}x)}$$

伝搬速度$v = \dfrac{1}{\sqrt{L'C'}}$

$+$**について** $\dfrac{dI_0}{dx} = i\omega\sqrt{L'C'}I_0 = -i\omega C' V_0$

より $-\sqrt{\dfrac{L'}{C'}}I_0^{\ -} = V_0^{\ -}$

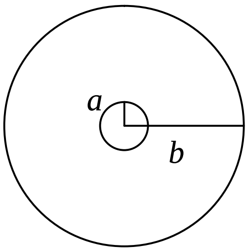
$-$**について** $\dfrac{dI_0}{dx} = -i\omega\sqrt{L'C'}I_0 = -i\omega C' V_0$

より$\sqrt{\dfrac{L'}{C'}}I_0^{\ +} = V_0^{\ +}$

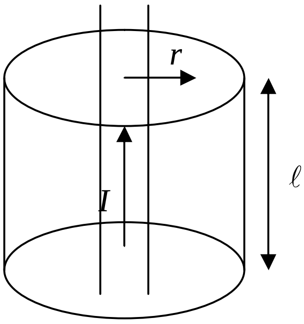
$Z_0 = \sqrt{\dfrac{L'}{C'}} = \sqrt{\dfrac{L}{C}}$ **特性インピーダンス**

$$I = I^+ + I^- = e^{i\omega t}(I_0^{\ +} + I_0^{\ -}) = A^+ e^{i\omega(t-\sqrt{L'C'}x)} + A^- e^{i\omega(t+\sqrt{L'C'}x)}$$

$$V = V^+ + V^- = e^{i\omega t}(V_0^{\ +} + V_0^{\ -}) = \sqrt{\frac{L'}{C'}}[A^+ e^{i\omega(t-\sqrt{L'C'}x)} - A^- e^{i\omega(t+\sqrt{L'C'}x)}]$$



断面　表皮効果で電流は表面のみを流れる



線電荷密度λ　$Q=\lambda\ell$

$2\pi r\ell E=\lambda\ell/\varepsilon$

$E=\dfrac{\lambda}{2\pi\varepsilon r}$　$V=\int_a^b Edr=\dfrac{\lambda}{2\pi\varepsilon}\ln\dfrac{b}{a}$

$C=\dfrac{Q}{V}=\dfrac{2\pi\varepsilon\ell}{\ln(b/a)}$　**単位長さ当たり**$C'=\dfrac{2\pi\varepsilon}{\ln(b/a)}$

$B=\dfrac{\mu I}{2\pi r}$

$\Phi_B=\int d\Phi_B=\int_a^b B\ell dr=\int_a^b\dfrac{\mu I}{2\pi r}\ell dr=\dfrac{\mu I}{2\pi}\ell\ln\dfrac{b}{a}$

$L=\dfrac{\Phi_B}{I}=\dfrac{\mu}{2\pi}\ell\ln\dfrac{b}{a}$　**単位長さ当たり**$L'=\dfrac{\mu}{2\pi}\ln\dfrac{b}{a}$

$L'C'=\varepsilon\mu$　$v=\dfrac{1}{\sqrt{\varepsilon\mu}}=\dfrac{c}{n}$

$$Z_0=\sqrt{\frac{L'}{C'}}=\sqrt{\frac{\frac{\mu}{2\pi}\ln\frac{b}{a}}{2\pi\varepsilon\Big/\ln\frac{b}{a}}}=\frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\frac{b}{a}$$

左から右への伝搬波 I^+,V^+　$V^+=Z_0I^+$

右から左への反射波I^-,V^-　$V^-=-Z_0I^-$

負荷Z_L**では**　$V_L=V^++V^-$　$I_L=I^++I^-$

$$V^++V^-=V_L=Z_LI_L=Z_L(I^++I^-)=\frac{Z_L}{Z_0}(V^+-V^-)$$

$\dfrac{V^-}{V^+}=\dfrac{Z_L-Z_0}{Z_L+Z_0}=r$　**電圧反射率**

$Z_L=Z_0$**とすれば（インピーダンス整合）、反射波は生じない**

特性インピーダンスによる反射の理解

真空の特性インピーダンス　真空中の電磁波の E と H の比

$$Z_0 = \frac{E_0}{H_0} = \frac{E_0}{B_0/\mu_0} = \frac{E_0}{(E_0/c)/\mu_0} = \mu_0 c = \mu_0 \frac{1}{\sqrt{\varepsilon_0\mu_0}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.7\Omega$$

ε, μの媒質との境界面での垂直入射による振幅反射率r

媒質の特性インピーダンス　$Z = \sqrt{\frac{\mu}{\varepsilon}}$　$Z_1 \to Z_2$

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{\sqrt{\frac{\mu_2}{\varepsilon_2}} - \sqrt{\frac{\mu_1}{\varepsilon_1}}}{\sqrt{\frac{\mu_2}{\varepsilon_2}} + \sqrt{\frac{\mu_1}{\varepsilon_1}}} = \frac{1 - \sqrt{\frac{\varepsilon_2}{\mu_2}}\sqrt{\frac{\mu_1}{\varepsilon_1}}}{1 + \sqrt{\frac{\varepsilon_2}{\mu_2}}\sqrt{\frac{\mu_1}{\varepsilon_1}}} = \frac{1 - \frac{\mu_1}{\mu_2}\sqrt{\frac{\varepsilon_2\mu_2}{\varepsilon_1\mu_1}}}{1 + \frac{\mu_1}{\mu_2}\sqrt{\frac{\varepsilon_2\mu_2}{\varepsilon_1\mu_1}}} = \frac{1-\beta}{1+\beta}$$

p.385 (9.82)**式**

ななめ入射

Eが入射面内にあるとき (P**偏光**)

$$Z_1 = \frac{E_1\cos\theta_I}{H_1} = \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_I \qquad Z_2 = \frac{E_2\cos\theta_T}{H_2} = \sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_T$$

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_T - \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_I}{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_T + \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_I} = \frac{\frac{\cos\theta_T}{\cos\theta_I} - \sqrt{\frac{\varepsilon_2}{\mu_2}}\sqrt{\frac{\mu_1}{\varepsilon_1}}}{\frac{\cos\theta_T}{\cos\theta_I} + \sqrt{\frac{\varepsilon_2}{\mu_2}}\sqrt{\frac{\mu_1}{\varepsilon_1}}} = \frac{\alpha - \frac{\mu_1}{\mu_2}\sqrt{\frac{\varepsilon_2\mu_2}{\varepsilon_1\mu_1}}}{\alpha + \frac{\mu_1}{\mu_2}\sqrt{\frac{\varepsilon_2\mu_2}{\varepsilon_1\mu_1}}} = \frac{\alpha-\beta}{\alpha+\beta}$$

p.390 (9.109)**式**

Bが入射面内にあるとき (S**偏光**)

$$Z_1 = \frac{E_1}{H_1\cos\theta_I} = \sqrt{\frac{\mu_1}{\varepsilon_1}}\frac{1}{\cos\theta_I} \qquad Z_2 = \frac{E_2}{H_2\cos\theta_T} = \sqrt{\frac{\mu_2}{\varepsilon_2}}\frac{1}{\cos\theta_T}$$

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{\sqrt{\frac{\mu_2}{\varepsilon_2}}\frac{1}{\cos\theta_T} - \sqrt{\frac{\mu_1}{\varepsilon_1}}\frac{1}{\cos\theta_I}}{\sqrt{\frac{\mu_2}{\varepsilon_2}}\frac{1}{\cos\theta_T} + \sqrt{\frac{\mu_1}{\varepsilon_1}}\frac{1}{\cos\theta_I}} = \frac{1 - \frac{\cos\theta_T}{\cos\theta_I}\sqrt{\frac{\varepsilon_2}{\mu_2}}\sqrt{\frac{\mu_1}{\varepsilon_1}}}{1 + \frac{\cos\theta_T}{\cos\theta_I}\sqrt{\frac{\varepsilon_2}{\mu_2}}\sqrt{\frac{\mu_1}{\varepsilon_1}}} = \frac{1 - \alpha\frac{\mu_1}{\mu_2}\sqrt{\frac{\varepsilon_2\mu_2}{\varepsilon_1\mu_1}}}{1 + \alpha\frac{\mu_1}{\mu_2}\sqrt{\frac{\varepsilon_2\mu_2}{\varepsilon_1\mu_1}}} = \frac{1-\alpha\beta}{1+\alpha\beta}$$