誘電率の分散の量子論（半古典論）　物質－量子力学　光－古典的電磁波

物質：原子（分子）の集団　原子（分子）波動関数

$\psi_m(r,t)=e^{-i\frac{E_m}{\hbar}t}\phi_m(r) \quad E_n=\hbar\omega_n$

ハミルトニアン$H=H_0+H'$　　H'：**光電場と物質との相互作用ハミルトニアン**

$H_0\phi_m=E_m\phi_m$

原子の双極子モーメント

$\boldsymbol{M}=-\sum_j e\boldsymbol{r}_j \quad \boldsymbol{r}_j$：$j$**番目の電子の座標**

$H'=-\boldsymbol{M}\cdot\boldsymbol{E}=-\boldsymbol{M}\cdot\boldsymbol{E}_0\cos\omega t=-\mu E_0\cos\omega t \quad \mu$：$\boldsymbol{M}$**の**$\boldsymbol{E}$**方向成分**

光の波長 >> 原子のサイズ → 原子内で光電場は一定　⇒　電気双極子近似

$\boldsymbol{M}_{mn}=\langle\phi_m|\boldsymbol{M}|\phi_n\rangle$**と**$\boldsymbol{E}_0$**のなす角を**$\theta$

$\mu_{mn}=\langle\phi_m|M\cos\theta|\phi_n\rangle=\langle\phi_m|M_x|\phi_n\rangle$

等方的物質の場合$\rightarrow\langle m|M_x|n\rangle=\langle m|M_y|n\rangle=\langle m|M_z|n\rangle$

$\therefore|\boldsymbol{M}_{mn}|^2=3|\mu_{mn}|^2$

$\Psi(r,t)=\sum_n C_n{}'(t)\psi_n(r,t)=\sum_n C_n{}'(t)e^{-i\frac{E_n}{\hbar}t}\phi_n(r)=\sum_n C_n(t)\phi_n(r) \qquad C_n(t)=C_n{}'(t)e^{-i\frac{E_n}{\hbar}t}$

$$H\Psi=i\hbar\frac{\partial\Psi}{\partial t}$$

$$(H_0+H')(\sum_n C_n\phi_n)=i\hbar\frac{\partial}{\partial t}(\sum_n C_n\phi_n)$$

$$\langle\phi_m|H_0+H'|\sum_n C_n\phi_n\rangle=\langle\phi_m|i\hbar\sum_n\frac{dC_n}{dt}\phi_n\rangle$$

$$C_mE_m+\sum_n C_n\langle\phi_m|H'|\phi_n\rangle=i\hbar\sum_n\frac{dC_n}{dt}\langle\phi_m|\phi_n\rangle$$

$$i\hbar\frac{dC_m}{dt}=E_mC_m+\sum_n C_nH'_{mn}$$

$$i\hbar\frac{dC_m(t)}{dt}=E_mC_m(t)+\sum_n C_n(t)(-\mu_{mn})E_0\cos\omega t$$

二準位系で、原子など中心対称性（反転対称性）がある場合を考える

ϕ_n：$even \quad \boldsymbol{r}\rightarrow-\boldsymbol{r}$**で波動関数が不変**　　$odd \quad \boldsymbol{r}\rightarrow-\boldsymbol{r}$**で符号のみ反転**

$\therefore\mu_{ii}=0$

$$\begin{cases} i\hbar\dfrac{dC_1}{dt}=E_1C_1-(\mu_{12}E_0\cos\omega t)C_2 \\ i\hbar\dfrac{dC_2}{dt}=E_2C_2-(\mu_{21}E_0\cos\omega t)C_1 \end{cases}$$

$$\begin{cases} \dfrac{dC_1}{dt}=-i\omega_1C_1+i(\dfrac{\mu_{12}E_0}{\hbar}\cos\omega t)C_2=-i\omega_1C_1+i(\xi\cos\omega t)C_2=-i\omega_1C_1+i\eta(t)C_2 \\ \dfrac{dC_2}{dt}=-i\omega_2C_2+i(\dfrac{\mu_{21}E_0}{\hbar}\cos\omega t)C_1=-i\omega_2C_2+i(\xi^*\cos\omega t)C_1=-i\omega_2C_2+i\eta^*(t)C_1 \end{cases}$$

$$\xi=\frac{\mu_{12}E_0}{\hbar} \quad \xi^*=\frac{\mu_{21}E_0}{\hbar} \quad \eta(t)=\xi\cos\omega t$$

密度行列

$\rho_{11} = C_1 C_1^* \quad \rho_{12} = C_1 C_2^*$

$\rho_{22} = C_2 C_2^* \quad \rho_{21} = C_2 C_1^*$

$$\frac{d\rho_{11}}{dt} = \frac{dC_1}{dt} C_1^* + C_1 \frac{dC_1^*}{dt} = (-i\omega_1 C_1 + i\eta C_2) C_1^* + C_1 (i\omega_1 C_1^* - i\eta^* C_2^*) = i(\eta\rho_{21} - \eta^* \rho_{12})$$

$$\frac{d\rho_{22}}{dt} = -\frac{d\rho_{11}}{dt} \quad \because \rho_{22} = 1 - \rho_{11}$$

$$\frac{d\rho_{12}}{dt} = \frac{dC_1}{dt} C_2^* + C_1 \frac{dC_2^*}{dt} = (-i\omega_1 C_1 + i\eta C_2) C_2^* + C_1 (i\omega_2 C_2^* - i\eta C_1^*)$$

$$= i(\omega_2 - \omega_1) C_1 C_2^* + i\eta (C_2 C_2^* - C_1 C_1^*)$$

$$= i\omega_0 \rho_{12} + i\eta(\rho_{22} - \rho_{11}) \qquad \omega_0 = \omega_2 - \omega_1$$

$$\frac{d\rho_{21}}{dt} = \frac{d\rho_{12}^*}{dt}$$

$$\frac{d\rho_{11}}{dt} = i(\eta\rho_{21} - \eta^* \rho_{12}) + \gamma_1 \rho_{22}$$

$$\frac{d\rho_{22}}{dt} = -i(\eta\rho_{21} - \eta^* \rho_{12}) - \gamma_1 \rho_{22}$$

$$\frac{d\rho_{12}}{dt} = i\omega_0 \rho_{12} + i\eta(\rho_{22} - \rho_{11}) - \gamma_2 \rho_{12}$$

$$\frac{d\rho_{21}}{dt} = -i\omega_0 \rho_{21} - i\eta^*(\rho_{22} - \rho_{11}) - \gamma_2 \rho_{21}$$

ρ_{12}の解

$\rho_{22} = 0, \rho_{11} = 1$

$$\frac{d\rho_{12}}{dt} - i\omega_0 \rho_{12} + \gamma_2 \rho_{12} = -i\eta = -i\xi \cos\omega t = -i\frac{\xi}{2}(e^{i\omega t} + e^{-i\omega t})$$

定常解$\rho_{12} = ae^{i\omega t} + be^{-i\omega t}$

$$[i(\omega - \omega_0) + \gamma_2] ae^{i\omega t} + [-i(\omega + \omega_0) + \gamma_2] be^{-i\omega t} = -i\frac{\xi}{2}(e^{i\omega t} + e^{-i\omega t})$$

$$\rho_{12} = \frac{-i\frac{\xi}{2} e^{i\omega t}}{i(\omega - \omega_0) + \gamma_2} + \frac{-i\frac{\xi}{2} e^{-i\omega t}}{-i(\omega + \omega_0) + \gamma_2} = \frac{\xi}{2}\left[\frac{e^{i\omega t}}{(\omega_0 - \omega) + i\gamma_2} + \frac{e^{-i\omega t}}{(\omega_0 + \omega) + i\gamma_2}\right]$$

巨視的分極

$$P(t) = \frac{N}{V}\langle \Psi | \mu | \Psi \rangle = \frac{N}{V}\langle C_1\phi_1 + C_2\phi_2 | \mu | C_1\phi_1 + C_2\phi_2 \rangle = \frac{N}{V}(C_1^* C_2 \mu_{12} + C_2^* C_1 \mu_{21}) = \frac{N}{V}(\rho_{21}\mu_{12} + \rho_{12}\mu_{21})$$

$$= \frac{N}{V}\left[\frac{\mu_{12}^* E_0}{2\hbar}\left(\frac{e^{-i\omega t}}{\omega_0 - \omega - i\gamma_2} + \frac{e^{i\omega t}}{\omega_0 + \omega - i\gamma_2}\right)\mu_{12} + \frac{\mu_{12} E_0}{2\hbar}\left(\frac{e^{i\omega t}}{\omega_0 - \omega + i\gamma_2} + \frac{e^{-i\omega t}}{\omega_0 + \omega + i\gamma_2}\right)\mu_{21}\right]$$

$$= \varepsilon_0 \frac{E_0}{2}\left[\frac{N}{V}\frac{|\mu_{12}|^2}{\varepsilon_0 \hbar}\left(\frac{1}{\omega_0 - \omega - i\gamma_2} + \frac{1}{\omega_0 + \omega + i\gamma_2}\right)e^{-i\omega t} + \frac{N}{V}\frac{|\mu_{12}|^2}{\varepsilon_0 \hbar}\left(\frac{1}{\omega_0 + \omega - i\gamma_2} + \frac{1}{\omega_0 - \omega + i\gamma_2}\right)e^{i\omega t}\right]$$

$$= \varepsilon_0 \frac{E_0}{2}\left[\chi(\omega)e^{-i\omega t} + \chi(-\omega)e^{i\omega t}\right] \qquad E(t) = \frac{E_0}{2}(e^{-i\omega t} + e^{i\omega t})$$

$$\chi(\omega) = \frac{N}{V}\frac{|\mu_{12}|^2}{\varepsilon_0 \hbar}\left(\frac{1}{\omega_0 - \omega - i\gamma_2} + \frac{1}{\omega_0 + \omega + i\gamma_2}\right)$$

$$\chi(\omega)=\frac{N}{V}\frac{|\mu_{12}|^2}{\varepsilon_0\hbar}\left(\frac{1}{\omega_0-\omega-i\gamma_2}+\frac{1}{\omega_0+\omega+i\gamma_2}\right)=\frac{N}{V}\frac{|\mu_{12}|^2}{\varepsilon_0\hbar}\frac{2\omega_0}{\omega_0{}^2-(\omega+i\gamma_2)^2}\quad \gamma_2=\Gamma_0/2$$

$$=\frac{N}{V}\frac{|\mu_{12}|^2}{\varepsilon_0\hbar}\frac{2\omega_0}{\omega_0{}^2-\omega^2+\Gamma_0{}^2/4-i\omega\Gamma_0}$$

$$=\frac{1}{\varepsilon_0}\frac{N}{V}\frac{e^2}{m}\frac{\frac{2m\omega_0|\mu_{12}|^2}{\hbar e^2}}{\omega_0{}^2-\omega^2+\Gamma_0{}^2/4-i\omega\Gamma_0}\quad\rightarrow\quad\frac{1}{\varepsilon_0}\frac{N}{V}\frac{e^2}{m}\sum_j\frac{f_{gj}}{\omega_{gj}{}^2-\omega^2-i\omega\Gamma_{gj}}$$

$$\omega_0{}^2+\Gamma_0{}^2/4\rightarrow\omega_0{}^2\quad \omega_0\rightarrow\omega_{gj}\quad \Gamma_0\rightarrow\Gamma_{gj}\quad f_{gj}=\frac{2m\omega_{gj}|\mu_{gj}|^2}{\hbar e^2}$$

f_{gj} ：振動子強度

振動子強度の総和則

$$\chi(\omega)=\frac{1}{\varepsilon_0}\frac{N}{V}\frac{e^2}{m}\sum_j\frac{f_{gj}}{\omega_{gj}{}^2-\omega^2-i\omega\Gamma_{gj}}$$

$\omega\rightarrow\infty\ (\omega>>\omega_{gj})$ **自由電子の振る舞いに近づく**

$$\chi(\omega)\rightarrow\frac{1}{\varepsilon_0}\frac{N}{V}\frac{e^2}{m}\Big(-\frac{1}{\omega^2}\sum_j f_{gj}\Big)\cdots(\mathrm{A})$$

自由電子の感受率

$$m\frac{d^2x}{dt^2}=-eE\quad x=\frac{eE}{m\omega^2}$$

$$p=-ex=-\frac{e^2E}{m\omega^2}$$

$$P=\frac{N}{V}p=-\frac{N}{V}\frac{e^2E}{m\omega^2}=\varepsilon_0\chi E$$

$$\chi=-\frac{1}{\varepsilon_0}\frac{N}{V}\frac{e^2}{m\omega^2}\cdots(\mathrm{B})$$

(A)**と**(B)**を比較して**

$\therefore\sum_j f_{gj}=1$　1**電子系**　N**電子系**$\sum_j f_{gj}=N$　**総和則**

水素原子の許容遷移　f_{gj}

1S－2P	121.567nm	0.4162
1S－3P	102.572nm	0.07910
1S－4P	97.254nm	0.02899

$A_{2P\text{-}1S}=6.26\times10^8\mathrm{s}^{-1}=\gamma_1=2\gamma_2$

Rabi 振動, Optical Stark 効果(交流 Stark 効果), 光章動(Optical nutation), Autler-Townes splitting

$$\boxed{\begin{aligned}
\frac{d\rho_{11}}{dt} &= i(\eta\rho_{21} - \eta^*\rho_{12}) - \Gamma_1\rho_{11} + \Gamma_2\rho_{22} \\
\frac{d\rho_{22}}{dt} &= -i(\eta\rho_{21} - \eta^*\rho_{12}) - \Gamma_2\rho_{22} + \Gamma_1\rho_{11} \\
\frac{d\rho_{12}}{dt} &= i\omega_0\rho_{12} + i\eta(\rho_{22} - \rho_{11}) - \gamma_2\rho_{12} \\
\frac{d\rho_{21}}{dt} &= -i\omega_0\rho_{21} - i\eta^*(\rho_{22} - \rho_{11}) - \gamma_2\rho_{21}
\end{aligned}}$$

$T \to 0 \quad \Gamma_2 \to \gamma_1 \quad \Gamma_1 \to 0$

$$\boxed{\begin{aligned}
\frac{d\rho_{11}}{dt} &= i(\eta\rho_{21} - \eta^*\rho_{12}) + \gamma_1\rho_{22} \\
\frac{d\rho_{22}}{dt} &= -i(\eta\rho_{21} - \eta^*\rho_{12}) - \gamma_1\rho_{22} \\
\frac{d\rho_{12}}{dt} &= i\omega_0\rho_{12} + i\eta(\rho_{22} - \rho_{11}) - \gamma_2\rho_{12} \\
\frac{d\rho_{21}}{dt} &= -i\omega_0\rho_{21} - i\eta^*(\rho_{22} - \rho_{11}) - \gamma_2\rho_{21}
\end{aligned}}$$

$$\boxed{\begin{aligned}
\frac{d\rho_{11}}{dt} &= \frac{i}{2}(\xi\rho_{21} - \xi^*\rho_{12})(e^{i\omega t} + e^{-i\omega t}) + \gamma_1\rho_{22} \\
\frac{d\rho_{22}}{dt} &= -\frac{i}{2}(\xi\rho_{21} - \xi^*\rho_{12})(e^{i\omega t} + e^{-i\omega t}) - \gamma_1\rho_{22} \\
\frac{d\rho_{12}}{dt} &= i\omega_0\rho_{12} + \frac{i}{2}\xi(\rho_{22} - \rho_{11})(e^{i\omega t} + e^{-i\omega t}) - \gamma_2\rho_{12} \\
\frac{d\rho_{21}}{dt} &= -i\omega_0\rho_{21} - \frac{i}{2}\xi^*(\rho_{22} - \rho_{11})(e^{i\omega t} + e^{-i\omega t}) - \gamma_2\rho_{21}
\end{aligned}}$$

$\tilde{\rho}_{12} = \rho_{12}e^{-i\omega t} \quad \tilde{\rho}_{21} = \rho_{21}e^{i\omega t} \quad \dfrac{d\tilde{\rho}_{12}}{dt} = \dfrac{d\rho_{12}}{dt}e^{-i\omega t} - i\omega\tilde{\rho}_{12}$

$$\boxed{\begin{aligned}
\frac{d\tilde{\rho}_{12}}{dt} &= i\omega_0\rho_{12}e^{-i\omega t} + \frac{i}{2}\xi(\rho_{22} - \rho_{11})(1 + e^{-i2\omega t}) - \gamma_2\rho_{12}e^{-i\omega t} - i\omega\tilde{\rho}_{12} \\
&\cong i(\omega_0 - \omega)\tilde{\rho}_{12} + \frac{i}{2}\xi(\rho_{22} - \rho_{11}) - \gamma_2\tilde{\rho}_{12} \qquad \text{回転波近似} \\
\frac{d\rho_{22}}{dt} &= -\frac{i}{2}(\xi\tilde{\rho}_{21}e^{-i\omega t} - \xi^*\tilde{\rho}_{12}e^{i\omega t})(e^{i\omega t} + e^{-i\omega t}) - \gamma_1\rho_{22} \\
&\cong -\frac{i}{2}(\xi\tilde{\rho}_{21} - \xi^*\tilde{\rho}_{12}) - \gamma_1\rho_{22} \\
\frac{d\tilde{\rho}_{21}}{dt} &= \frac{d\tilde{\rho}_{12}{}^*}{dt} \cong -i(\omega_0 - \omega)\tilde{\rho}_{21} - \frac{i}{2}\xi^*(\rho_{22} - \rho_{11}) - \gamma_2\tilde{\rho}_{21} \\
\frac{d\rho_{11}}{dt} &= -\frac{d\rho_{22}}{dt} \cong \frac{i}{2}(\xi\tilde{\rho}_{21} - \xi^*\tilde{\rho}_{12}) + \gamma_1\rho_{22}
\end{aligned}}$$

$\rho_{11} = \rho_{11}{}^{(0)}e^{\lambda t} \quad \rho_{22} = \rho_{22}{}^{(0)}e^{\lambda t} \quad \tilde{\rho}_{12} = \tilde{\rho}_{12}{}^{(0)}e^{\lambda t} \quad \tilde{\rho}_{21} = \tilde{\rho}_{21}{}^{(0)}e^{\lambda t}$

$$\rho_{11}=\rho_{11}{}^{(0)}e^{\lambda t}\quad \rho_{22}=\rho_{22}{}^{(0)}e^{\lambda t}\quad \tilde{\rho}_{12}=\tilde{\rho}_{12}{}^{(0)}e^{\lambda t}\quad \tilde{\rho}_{21}=\tilde{\rho}_{21}{}^{(0)}e^{\lambda t}$$

$$\begin{pmatrix} -\lambda & \gamma_1 & -\frac{i}{2}\xi^* & \frac{i}{2}\xi \\ 0 & -\lambda-\gamma_1 & \frac{i}{2}\xi^* & -\frac{i}{2}\xi \\ -\frac{i}{2}\xi & \frac{i}{2}\xi & -\lambda+i(\omega_0-\omega)-\gamma_2 & 0 \\ \frac{i}{2}\xi^* & -\frac{i}{2}\xi^* & 0 & -\lambda-i(\omega_0-\omega)-\gamma_2 \end{pmatrix}\begin{pmatrix}\rho_{11}{}^{(0)}\\ \rho_{22}{}^{(0)}\\ \tilde{\rho}_{12}{}^{(0)}\\ \tilde{\rho}_{21}{}^{(0)}\end{pmatrix}=0$$

$\gamma_1=\gamma_2=0$**のとき** $\lambda^2[\lambda^2+|\xi|^2+(\omega_0-\omega)^2]=0 \quad \lambda=0,\pm i\sqrt{|\xi|^2+(\omega_0-\omega)^2}\,(\equiv\pm\Omega i)$

$$\rho_{12}(t)=\tilde{\rho}_{12}(t)e^{i\omega t}=(Ae^{i\Omega t}+Be^{-i\Omega t}+C)e^{i\omega t}$$

$$\rho_{22}(t)=\alpha e^{i\Omega t}+\beta e^{-i\Omega t}+\gamma$$

初期条件 $\rho_{12}(0)=0 \quad \rho_{11}(0)=1 \quad \rho_{22}(0)=0 \quad \left(\frac{d\tilde{\rho}_{12}}{dt}\right)_{t=0}=-\frac{i}{2}\xi \quad \left(\frac{d\rho_{22}}{dt}\right)_{t=0}=0$

$$\left(\frac{d^2\tilde{\rho}_{12}}{dt^2}\right)_{t=0}=i(\omega_0-\omega)\frac{d\tilde{\rho}_{12}}{dt}+\frac{i}{2}\xi\left(\frac{d\rho_{22}}{dt}-\frac{d\rho_{11}}{dt}\right)=\frac{\xi}{2}(\omega_0-\omega)$$

$$\left(\frac{d^2\rho_{22}}{dt^2}\right)_{t=0}=-\frac{i}{2}\left(\xi\frac{d\tilde{\rho}_{21}}{dt}-\xi^*\frac{d\tilde{\rho}_{12}}{dt}\right)=-\frac{i}{2}\left[\xi\frac{i}{2}\xi^*-\xi^*\left(-\frac{i}{2}\xi\right)\right]=-\frac{i}{2}\cdot i\,|\xi|^2=\frac{|\xi|^2}{2}$$

$$A+B+C=0 \quad i\Omega(A-B)=-\frac{i}{2}\xi \quad -\Omega^2(A+B)=\frac{\xi}{2}(\omega_0-\omega)$$

$$\alpha+\beta+\gamma=0 \quad i\Omega(\alpha-\beta)=0 \quad -\Omega^2(\alpha+\beta)=\frac{|\xi|^2}{2}$$

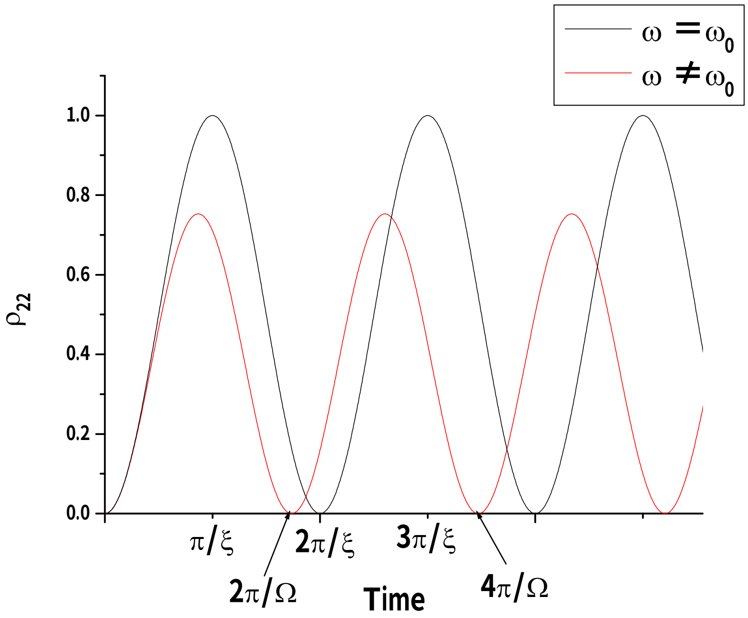
$$A=-\frac{\xi}{4\Omega^2}[\Omega+(\omega_0-\omega)] \quad B=\frac{\xi}{4\Omega^2}[\Omega-(\omega_0-\omega)] \quad C=\frac{\xi}{2\Omega^2}(\omega_0-\omega)$$

$$\alpha=-\frac{|\xi|^2}{4\Omega^2} \quad \beta=-\frac{|\xi|^2}{4\Omega^2} \quad \gamma=\frac{|\xi|^2}{2\Omega^2}$$

$$\rho_{22}(t)=-\frac{|\xi|^2}{4\Omega^2}(e^{i\Omega t}+e^{-i\Omega t}-2)=-\frac{|\xi|^2}{4\Omega^2}(2\cos\Omega t-2)=\frac{|\xi|^2}{\Omega^2}\frac{1-\cos\Omega t}{2}=\frac{|\xi|^2}{\Omega^2}\sin^2\frac{\Omega}{2}t$$

$\Omega=\sqrt{|\xi|^2+(\omega_0-\omega)^2}$: Rabi**周波数**

For $\omega=\omega_0$, $\Omega=|\xi|$ $\quad \rho_{22}(t)=\sin^2\frac{|\xi|}{2}t$



振動が観測できるためには $\omega=\omega_0$ **で** $|\xi|>>3\gamma_2$

水素原子　1s⇔2p　$3\gamma_2 \sim 10^9\,\mathrm{s}^{-1}$　$|\xi|=3\gamma_2$　$E_0 \sim 10^5\,\mathrm{V/m}$　$I \sim 10^3\,\mathrm{W/cm}^2$

Na **の** D **線**　589nm　**自然幅**～10MHz＝$10^7\,\mathrm{s}^{-1}$　$E_0 \sim 10^3\,\mathrm{V/m}$　$I \sim 0.1\,\mathrm{W/cm}^2$

地上の太陽光(真夏、晴天)　$I \approx 1\,\mathrm{kW/m}^2 = 0.1\mathrm{W/cm}^2$

水素原子　1s⇔2p　$|\xi|=\omega_0$ **のためには、** $E_0 = 3\times10^{11}\,\mathrm{V/m}$ **の光電場が必要**

分極は？

$$\rho_{12}(t)=\tilde{\rho}_{12}(t)e^{i\omega t}=(Ae^{i\Omega t}+Be^{-i\Omega t}+C)e^{i\omega t}$$

$$A=-\frac{\xi}{4\Omega^2}[\Omega+(\omega_0-\omega)]\quad B=\frac{\xi}{4\Omega^2}[\Omega-(\omega_0-\omega)]\quad C=\frac{\xi}{2\Omega^2}(\omega_0-\omega)$$

$$\rho_{12}(t)=\left\{\frac{\xi}{4\Omega^2}[-\Omega-(\omega_0-\omega)]e^{i\Omega t}+\frac{\xi}{4\Omega^2}[\Omega-(\omega_0-\omega)]e^{-i\Omega t}+\frac{\xi}{2\Omega^2}(\omega_0-\omega)\right\}e^{i\omega t}$$

$$=\left\{\frac{\xi}{4\Omega}[-e^{i\Omega t}+e^{-i\Omega t})]-\frac{\xi(\omega_0-\omega)}{4\Omega^2}(e^{i\Omega t}+e^{-i\Omega t})+\frac{\xi(\omega_0-\omega)}{2\Omega^2}\right\}e^{i\omega t}$$

$$=\left\{\frac{\xi}{2\Omega}(-i\sin\Omega t)+\frac{\xi(\omega_0-\omega)}{2\Omega^2}(1-\cos\Omega t)\right\}e^{i\omega t}$$

$$P(t)=\frac{N}{V}(\rho_{12}\mu_{21}+\rho_{21}\mu_{12})$$

$$=\frac{N}{V}\frac{|\mu_{12}|^2E_0}{\Omega\hbar}\left\{\left[(-\frac{i}{2}\sin\Omega t)e^{i\omega t}+(\frac{i}{2}\sin\Omega t)e^{-i\omega t}\right]+\frac{(\omega_0-\omega)}{2\Omega}(1-\cos\Omega t)(e^{i\omega t}+e^{-i\omega t})\right\}$$

$$=\frac{N}{V}\frac{|\mu_{12}|^2E_0}{\Omega\hbar}\left\{\sin\Omega t\sin\omega t+\frac{\omega_0-\omega}{\Omega}(1-\cos\Omega t)\cos\omega t\right\}$$

For $\omega=\omega_0$,　$P(t)=\frac{N}{V}\frac{|\mu_{12}|^2E_0}{\Omega\hbar}\sin\xi t\sin\omega t$

2準位系、中心対称、回転波近似、$\gamma_1, \gamma_2 = 0$のもとで正しい式

$$\rho_{22}(t) = \frac{|\xi|^2}{\Omega^2}\sin^2\frac{\Omega}{2}t$$

$$\rho_{22}(t) \quad \to \quad |\xi|^2 \frac{\sin^2\frac{(\omega_0-\omega)}{2}t}{(\omega_0-\omega)^2} \quad (\omega_0-\omega)^2 >> |\xi|^2$$

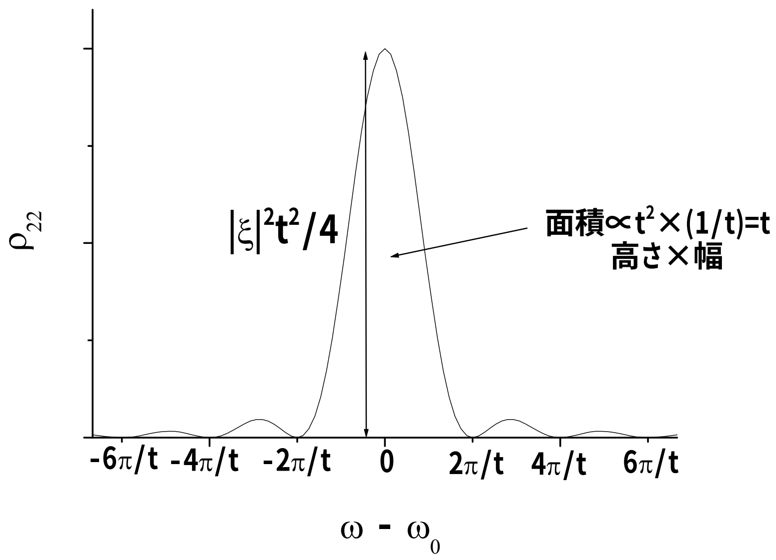
$\Omega = \sqrt{|\xi|^2 + (\omega_0-\omega)^2}$: Rabi周波数

$\Omega t << 1\ (\rho_{22} << 1)$の条件で、

$\omega \to \omega_0$のとき、

$$\rho_{22}(t) \quad \to \quad \frac{|\xi|^2}{4}t^2$$

$\omega = \omega_0$のとき、$(\omega_0-\omega)^2 >> |\xi|^2$ が成り立たないが、$\rho_{22}(t) = \sin^2\frac{|\xi|}{2}t \to \frac{|\xi|^2}{4}t^2$でOK



$$\rho_{22}(t) \quad = \quad \frac{|\xi|^2}{2}t\frac{\sin^2((\omega_0-\omega)t/2)}{(\omega_0-\omega)^2 t/2} \quad \frac{1}{\pi}\lim_{t\to\infty}\frac{\sin^2((\omega_0-\omega)t/2)}{(\omega_0-\omega)^2 t/2} \to \delta(\omega_0-\omega)\text{より}$$

$\rho_{22}(t) = \frac{\pi|\xi|^2}{2}t\delta(\omega_0-\omega)$ 遷移確率$w_{12} = \frac{d\rho_{22}}{dt} = \frac{\pi}{2\hbar^2}|\mu_{12}E_0|^2\ \delta(\omega_0-\omega)$ Fermiの黄金律

$$\frac{\varepsilon_0}{2}E_0{}^2 = \int U(\omega)d\omega$$

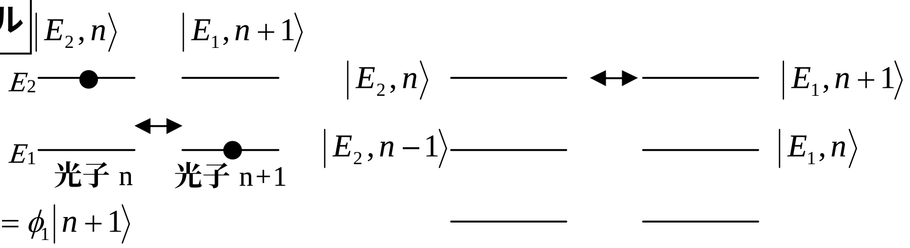
$$\rho_{22}(t) = \frac{|\xi|^2}{4}\frac{\sin^2\left(\frac{\omega_0-\omega}{2}t\right)}{\left(\frac{\omega_0-\omega}{2}\right)^2} = \frac{1}{2\varepsilon_0}\frac{|\mu_{12}|^2}{\hbar^2}\frac{\varepsilon_0 E_0{}^2}{2}\frac{\sin^2\left(\frac{\omega_0-\omega}{2}t\right)}{\left(\frac{\omega_0-\omega}{2}\right)^2} = \frac{1}{2\varepsilon_0}\frac{|\mu_{12}|^2}{\hbar^2}\int d\omega U(\omega)\frac{\sin^2\left(\frac{\omega_0-\omega}{2}t\right)}{\left(\frac{\omega_0-\omega}{2}\right)^2}$$

$$= \frac{1}{2\varepsilon_0}\frac{|\mu_{12}|^2}{\hbar^2}U(\omega_0)\int d\omega\frac{\sin^2\left(\frac{\omega_0-\omega}{2}t\right)}{\left(\frac{\omega_0-\omega}{2}\right)^2} = \frac{1}{2\varepsilon_0}\frac{|\mu_{12}|^2}{\hbar^2}U(\omega_0)\cdot 2t\int_{-\infty}^{\infty}dx\frac{\sin^2 x}{x^2} = \frac{1}{2\varepsilon_0}\frac{|\mu_{12}|^2}{\hbar^2}U(\omega_0)\cdot 2\pi t$$

遷移確率$w_{12} = \frac{d\rho_{22}(t)}{dt} = \frac{\pi|\mu_{12}|^2}{\varepsilon_0\hbar^2}U(\omega_0) \equiv B_{12}U(\omega_0)$ $\quad B_{12} = \frac{\pi|\mu_{12}|^2}{\varepsilon_0\hbar^2}[\frac{\mathrm{m}^3}{\mathrm{J\cdot s^2}}]$ EinsteinのB係数 誘導吸収･放出

Rabi 振動を理解するための

$\boxed{\text{dressed atom model 結合量子井戸モデル}}$



$\hbar\omega_0 = E_2 - E_1$

$$\begin{pmatrix} E_2 + n\hbar\omega & 0 \\ 0 & E_1 + (n+1)\hbar\omega \end{pmatrix}$$

$|E_1, n+1\rangle$ **電子状態**E_1, $n+1$**光子** $\psi_1 = \phi_1|n+1\rangle$

$\updownarrow \dfrac{\mu_{12}E_0}{2}$ **相互作用**

$|E_2, n\rangle$ **電子状態**E_2, n**光子** $\psi_2 = \phi_2|n\rangle$

$$\begin{vmatrix} E_2 + n\hbar\omega - \lambda & \dfrac{\mu_{12}E_0}{2} \\ \dfrac{\mu_{12}{}^{*}E_0{}^{*}}{2} & E_1 + (n+1)\hbar\omega - \lambda \end{vmatrix} = (E_2 + n\hbar\omega - \lambda)(E_1 + (n+1)\hbar\omega - \lambda) - \frac{|\mu_{12}E_0|^2}{4}$$

$$= \lambda^2 - [E_1 + E_2 + (2n+1)\hbar\omega]\lambda + (E_2 + n\hbar\omega)(E_1 + (n+1)\hbar\omega) - \frac{|\mu_{12}E_0|^2}{4} = 0$$

固有値$\lambda = \dfrac{1}{2}[E_1 + E_2 + (2n+1)\hbar\omega \pm \sqrt{(E_1 - E_2 + \hbar\omega)^2 + |\mu_{12}E_0|^2}\,]$

$\hbar\omega_0 = E_2 - E_1 \quad \hbar\Delta = E_1 + E_2 + (2n+1)\hbar\omega \quad \delta = \omega_0 - \omega \quad \xi = \dfrac{\mu_{12}E_0}{\hbar} \quad \Omega = \sqrt{\delta^2 + |\xi|^2}$

$$\lambda_{\pm} = \frac{1}{2}\hbar(\Delta \pm \Omega)$$

固有状態Ψ_+, Ψ_-

$$\lambda = \lambda_+ \quad \begin{pmatrix} \frac{1}{2}\hbar(\delta - \Omega) & \frac{1}{2}\hbar\xi \\ \frac{1}{2}\hbar\xi^{*} & \frac{1}{2}\hbar(-\delta - \Omega) \end{pmatrix}\begin{pmatrix} x_+ \\ y_+ \end{pmatrix} = 0 \quad \begin{pmatrix} x_+ \\ y_+ \end{pmatrix} = \frac{1}{\sqrt{|\xi|^2 + (\Omega - \delta)^2}}\begin{pmatrix} \xi \\ \Omega - \delta \end{pmatrix} \equiv \Psi_+$$

$$\lambda = \lambda_- \quad \begin{pmatrix} \frac{1}{2}\hbar(\delta + \Omega) & \frac{1}{2}\hbar\xi \\ \frac{1}{2}\hbar\xi^{*} & \frac{1}{2}\hbar(-\delta + \Omega) \end{pmatrix}\begin{pmatrix} x_- \\ y_- \end{pmatrix} = 0 \quad \begin{pmatrix} x_- \\ y_- \end{pmatrix} = \frac{1}{\sqrt{|\xi|^2 + (\Omega + \delta)^2}}\begin{pmatrix} \xi \\ -\Omega - \delta \end{pmatrix} \equiv \Psi_-$$

$$\Psi_+ = \frac{1}{\sqrt{|\xi|^2 + (\Omega - \delta)^2}}[\xi\psi_2 + (\Omega - \delta)\psi_1]$$

$$\Psi_- = \frac{1}{\sqrt{|\xi|^2 + (\Omega + \delta)^2}}[\xi\psi_2 - (\Omega + \delta)\psi_1]$$

任意の状態Ψ**は**$\Psi_+ e^{-i\frac{\lambda_+}{\hbar}t}, \Psi_- e^{-i\frac{\lambda_-}{\hbar}t}$**の線形結合で表せる**

$$\Psi = C_1 e^{-i\frac{\lambda_+}{\hbar}t}\Psi_+ + C_2 e^{-i\frac{\lambda_-}{\hbar}t}\Psi_- = C_1 e^{-i\frac{\Delta+\Omega}{2}t}\Psi_+ + C_2 e^{-i\frac{\Delta-\Omega}{2}t}\Psi_-$$

$$= e^{-i\frac{\Delta}{2}t}[C_1 e^{-i\frac{\Omega}{2}t}\Psi_+ + C_2 e^{i\frac{\Omega}{2}t}\Psi_-]$$

$$= e^{-i\frac{\Delta}{2}t}\left[C_1 e^{-i\frac{\Omega}{2}t}\frac{\xi\psi_2 + (\Omega - \delta)\psi_1}{\sqrt{|\xi|^2 + (\Omega - \delta)^2}} + C_2 e^{i\frac{\Omega}{2}t}\frac{\xi\psi_2 - (\Omega + \delta)\psi_1}{\sqrt{|\xi|^2 + (\Omega + \delta)^2}}\right]$$

$$= e^{-i\frac{\Delta}{2}t}\left[\left(\frac{\xi C_1 e^{-i\frac{\Omega}{2}t}}{\sqrt{|\xi|^2 + (\Omega - \delta)^2}} + \frac{\xi C_2 e^{i\frac{\Omega}{2}t}}{\sqrt{|\xi|^2 + (\Omega + \delta)^2}}\right)\psi_2 + \left(\frac{(\Omega - \delta)C_1 e^{-i\frac{\Omega}{2}t}}{\sqrt{|\xi|^2 + (\Omega - \delta)^2}} - \frac{(\Omega + \delta)C_2 e^{i\frac{\Omega}{2}t}}{\sqrt{|\xi|^2 + (\Omega + \delta)^2}}\right)\psi_1\right]$$

初期条件$t=0$**で**$\Psi=\psi_1=\phi_1|n+1\rangle$

$$\frac{\xi C_1}{\sqrt{|\xi|^2+(\Omega-\delta)^2}}+\frac{\xi C_2}{\sqrt{|\xi|^2+(\Omega+\delta)^2}}=0$$

$$\frac{(\Omega-\delta)C_1}{\sqrt{|\xi|^2+(\Omega-\delta)^2}}-\frac{(\Omega+\delta)C_2}{\sqrt{|\xi|^2+(\Omega+\delta)^2}}=1$$

$$|\xi|^2+(\Omega-\delta)^2=|\xi|^2+\Omega^2-2\Omega\delta+\delta^2=2\Omega^2-2\Omega\delta=2\Omega(\Omega-\delta)$$

$$|\xi|^2+(\Omega+\delta)^2=|\xi|^2+\Omega^2+2\Omega\delta+\delta^2=2\Omega^2+2\Omega\delta=2\Omega(\Omega+\delta)$$

$$\frac{C_1}{\sqrt{\Omega-\delta}}+\frac{C_2}{\sqrt{\Omega+\delta}}=0$$

$$\sqrt{\Omega-\delta}\,C_1-\sqrt{\Omega+\delta}\,C_2=\sqrt{2\Omega}$$

$$C_1=\sqrt{\frac{\Omega-\delta}{2\Omega}}\quad C_2=-\sqrt{\frac{\Omega+\delta}{2\Omega}}$$

$$\Psi=e^{-i\frac{\Delta}{2}t}\left[\left(\sqrt{\frac{\Omega-\delta}{2\Omega}}\frac{\xi e^{-i\frac{\Omega}{2}t}}{\sqrt{2\Omega(\Omega-\delta)}}-\sqrt{\frac{\Omega+\delta}{2\Omega}}\frac{\xi e^{i\frac{\Omega}{2}t}}{\sqrt{2\Omega(\Omega+\delta)}}\right)\psi_2+\left(\sqrt{\frac{\Omega-\delta}{2\Omega}}\frac{(\Omega-\delta)e^{-i\frac{\Omega}{2}t}}{\sqrt{2\Omega(\Omega-\delta)}}+\sqrt{\frac{\Omega+\delta}{2\Omega}}\frac{(\Omega+\delta)e^{i\frac{\Omega}{2}t}}{\sqrt{2\Omega(\Omega+\delta)}}\right)\psi_1\right]$$

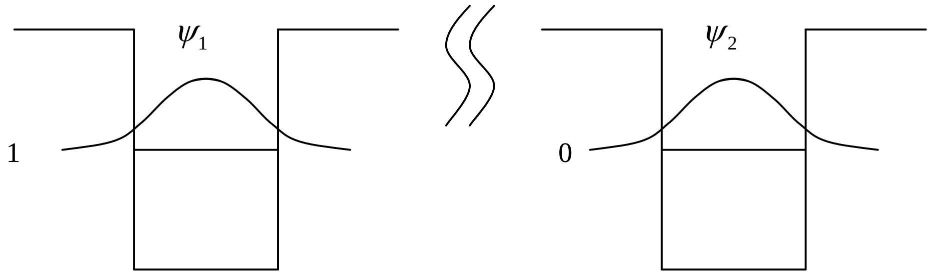
$$=e^{-i\frac{\Delta}{2}t}\left[\left(\frac{\xi e^{-i\frac{\Omega}{2}t}}{2\Omega}-\frac{\xi e^{i\frac{\Omega}{2}t}}{2\Omega}\right)\psi_2+\left(\frac{(\Omega-\delta)e^{-i\frac{\Omega}{2}t}}{2\Omega}+\frac{(\Omega+\delta)e^{i\frac{\Omega}{2}t}}{2\Omega}\right)\psi_1\right]$$

$$=e^{-i\frac{\Delta}{2}t}\left[\left(-i\frac{\xi}{\Omega}\sin\frac{\Omega}{2}t\right)\psi_2+\left(\cos\frac{\Omega}{2}t+i\frac{\delta}{\Omega}\sin\frac{\Omega}{2}t\right)\psi_1\right]$$

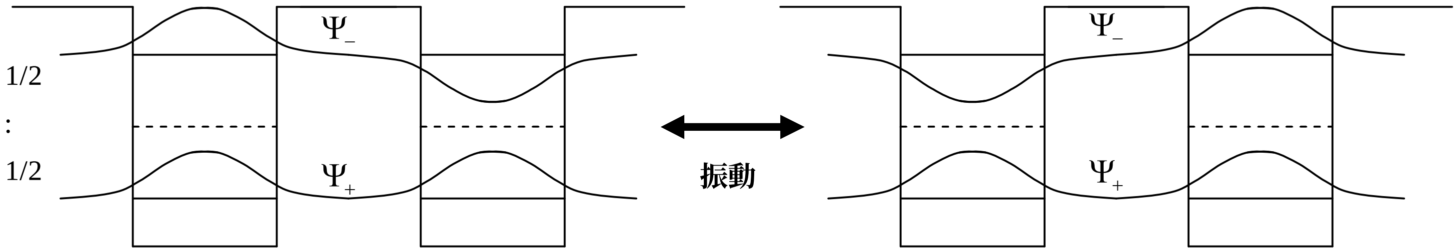
$\psi_2=\phi_2|n\rangle$**の確率**$\dfrac{|\xi|^2}{\Omega^2}\sin^2\dfrac{\Omega}{2}t$　Rabi**振動**

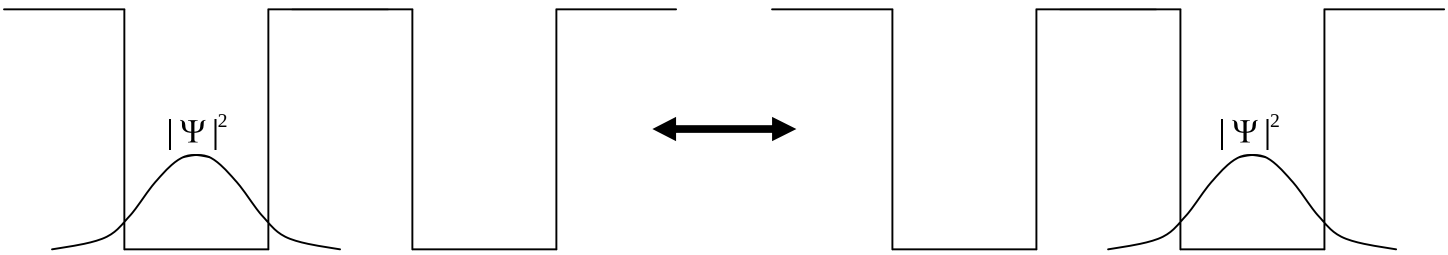
共鳴のとき、 $\omega = \omega_0 \quad E_1 + (n+1)\hbar\omega = E_2 + n\hbar\omega$

相互作用なし



相互作用あり

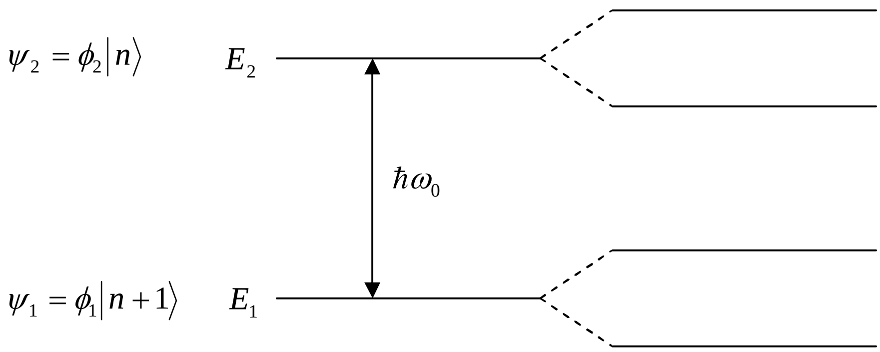




コヒーレント振動

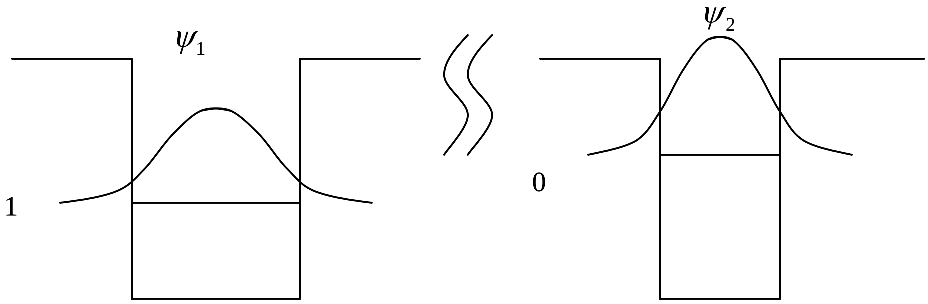
ここで相互作用 off **にすると電子が** $|E_2\rangle$ **に残る**

エネルギー保存（実励起）

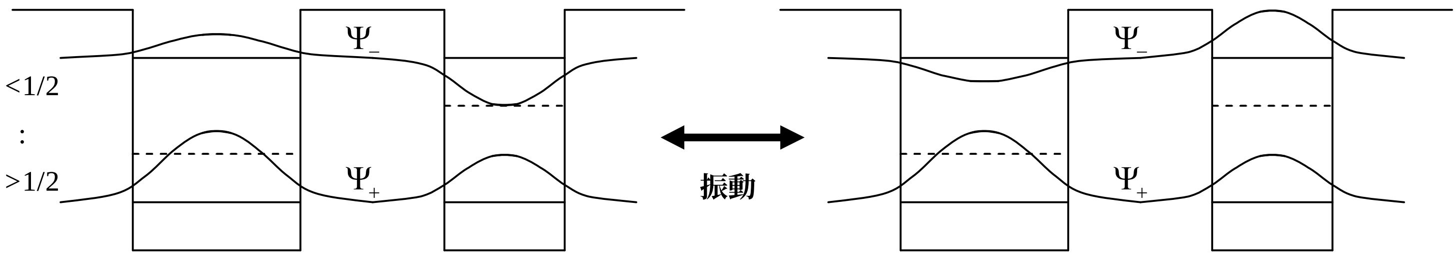


非共鳴のとき、 $\omega < \omega_0 \quad E_1 + (n+1)\hbar\omega < E_2 + n\hbar\omega$

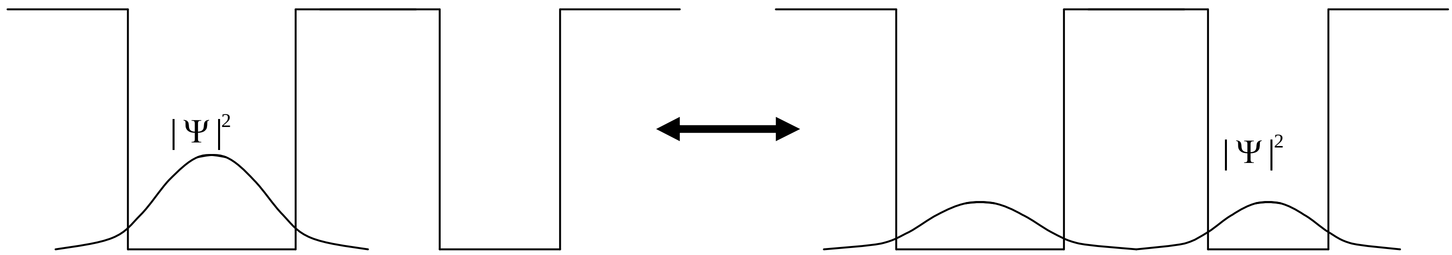
相互作用なし



相互作用あり



エネルギーは保存

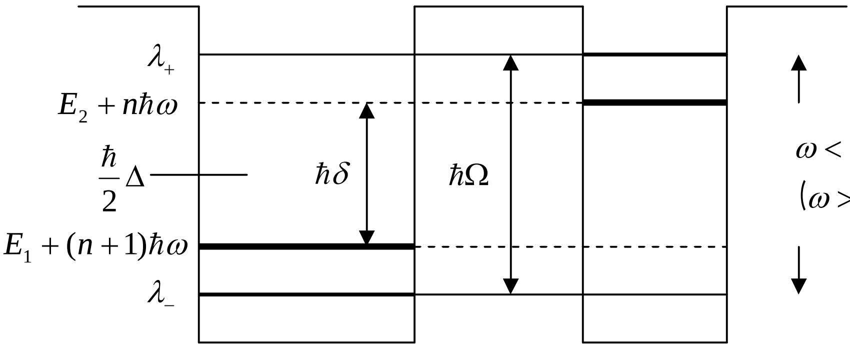


コヒーレント振動

ここで相互作用 off にするとエネルギー保存しないので電子は $|E_1\rangle$ に戻る（仮想励起）

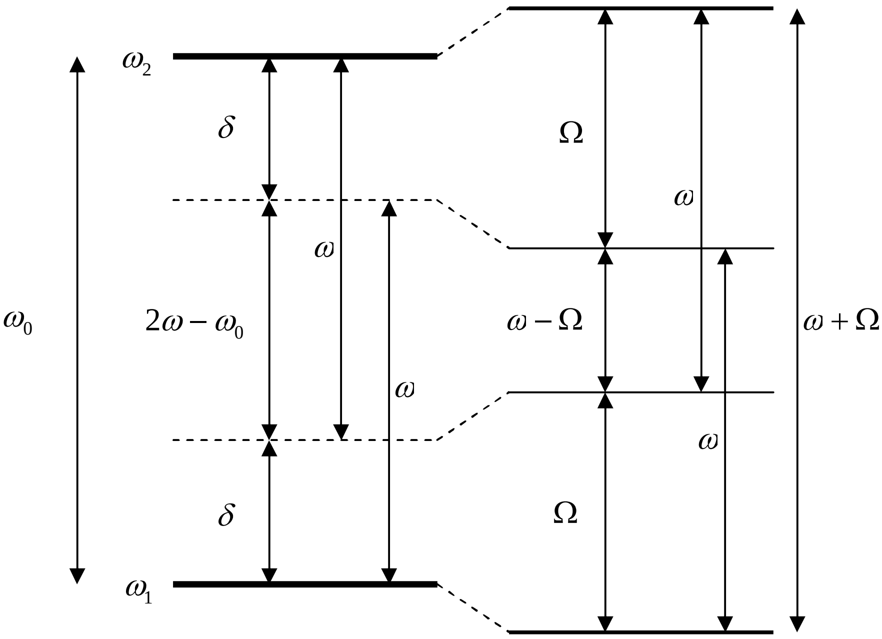
$\omega < \omega_0 \quad E_1 + (n+1)\hbar\omega < E_2 + n\hbar\omega \quad \delta = \omega_0 - \omega \quad \Omega = \sqrt{\delta^2 + |\xi|^2} \quad \hbar\Delta = E_1 + E_2 + (2n+1)\hbar\omega$

$\lambda_\pm = \dfrac{\hbar}{2}(\Delta \pm \Omega)$



$\omega < \omega_0$のとき、吸収はブルーシフト

($\omega > \omega_0$のとき、吸収はレッドシフト)



Optical Bloch 方程式

$$\frac{d\rho_{11}}{dt}=i(\eta\rho_{21}-\eta^*\rho_{12})-\Gamma_1\rho_{11}+\Gamma_2\rho_{22}$$
$$\frac{d\rho_{22}}{dt}=-i(\eta\rho_{21}-\eta^*\rho_{12})-\Gamma_2\rho_{22}+\Gamma_1\rho_{11}$$
$$\frac{d\rho_{12}}{dt}=i\omega_0\rho_{12}+i\eta(\rho_{22}-\rho_{11})-\gamma_2\rho_{12}$$
$$\frac{d\rho_{21}}{dt}=-i\omega_0\rho_{21}-i\eta^*(\rho_{22}-\rho_{11})-\gamma_2\rho_{21}$$

詳細平衡 detailed balance

$$\frac{\rho_{22}}{\rho_{11}}=\exp\left(-\frac{E_2-E_1}{kT}\right)=e^{-\frac{\hbar\omega_0}{kT}}$$

$$\rho_{11}\underset{\Gamma_1}{\rightarrow}\rho_{22}\qquad \rho_{11}\underset{\Gamma_2}{\leftarrow}\rho_{22}$$

$$\frac{d\rho_{11}}{dt}=-\Gamma_1\rho_{11}+\Gamma_2\rho_{22}=0\qquad \frac{d\rho_{22}}{dt}=-\Gamma_2\rho_{22}+\Gamma_1\rho_{11}=0$$

$\frac{\rho_{22}}{\rho_{11}}=\frac{\Gamma_1}{\Gamma_2}=e^{-\frac{\hbar\omega_0}{kT}}$ $T\approx 0$ では $\Gamma_1<<\Gamma_2$

$$\rho_{11}+\rho_{22}=1\quad \rho_{11}=\rho\quad \rho_{22}=\rho e^{-\frac{\hbar\omega_0}{kT}}\quad \rho_{11}+\rho_{22}=\rho(1+e^{-\frac{\hbar\omega_0}{kT}})=1\quad \rho=\frac{1}{1+e^{-\frac{\hbar\omega_0}{kT}}}$$

$$W\equiv\rho_{22}-\rho_{11}\quad W_{t\to\infty}=\rho e^{-\frac{\hbar\omega_0}{kT}}-\rho=\rho(e^{-\frac{\hbar\omega_0}{kT}}-1)=\frac{e^{-\frac{\hbar\omega_0}{kT}}-1}{e^{-\frac{\hbar\omega_0}{kT}}+1}\to -1\,(T\to 0)\quad \frac{\Gamma_1}{\Gamma_2}\to 0\,(T\to 0)$$

$$W-W_\infty=\rho_{22}-\rho_{11}-\frac{e^{-\frac{\hbar\omega_0}{kT}}-1}{e^{-\frac{\hbar\omega_0}{kT}}+1}$$

$$(e^{-\frac{\hbar\omega_0}{kT}}+1)(W-W_\infty)=(e^{-\frac{\hbar\omega_0}{kT}}+1)(1-2\rho_{11})-(e^{-\frac{\hbar\omega_0}{kT}}-1)$$
$$=2-2\rho_{11}(e^{-\frac{\hbar\omega_0}{kT}}+1)$$

$$-2\Gamma_2\rho_{22}+2\Gamma_1\rho_{11}=-2\Gamma_2(\rho_{22}-\frac{\Gamma_1}{\Gamma_2}\rho_{11})=-2\Gamma_2(1-\rho_{11}-e^{-\frac{\hbar\omega_0}{kT}}\rho_{11})$$
$$=-2\Gamma_2[1-\rho_{11}(1+e^{-\frac{\hbar\omega_0}{kT}})]=-\Gamma_2[2-2\rho_{11}(1+e^{-\frac{\hbar\omega_0}{kT}})]=-\Gamma_2(e^{-\frac{\hbar\omega_0}{kT}}+1)(W-W_\infty)$$

$$\frac{dW}{dt}=[-i(\eta\rho_{21}-\eta^*\rho_{12})-\Gamma_2\rho_{22}+\Gamma_1\rho_{11}]-[i(\eta\rho_{21}-\eta^*\rho_{12})-\Gamma_1\rho_{11}+\Gamma_2\rho_{22}]$$
$$=-i\,2(\eta\rho_{21}-\eta^*\rho_{12})-2\Gamma_2\rho_{22}+2\Gamma_1\rho_{11}$$
$$=-i\,2(\eta\rho_{21}-\eta^*\rho_{12})-\Gamma_2(e^{-\frac{\hbar\omega_0}{kT}}+1)(W-W_\infty)$$
$$=-i\,2(\eta\rho_{21}-\eta^*\rho_{12})-\gamma_1(W-W_\infty)$$

$$u\equiv\rho_{12}+\rho_{21}$$
$$v=i(\rho_{21}-\rho_{12})$$

$$\frac{d\rho_{11}}{dt} = i(\eta\rho_{21} - \eta^*\rho_{12}) - \Gamma_1\rho_{11} + \Gamma_2\rho_{22}$$

$$\frac{d\rho_{22}}{dt} = -i(\eta\rho_{21} - \eta^*\rho_{12}) - \Gamma_2\rho_{22} + \Gamma_1\rho_{11}$$

$$\frac{d\rho_{12}}{dt} = i\omega_0\rho_{12} + i\eta(\rho_{22} - \rho_{11}) - \gamma_2\rho_{12}$$

$$\frac{d\rho_{21}}{dt} = -i\omega_0\rho_{21} - i\eta^*(\rho_{22} - \rho_{11}) - \gamma_2\rho_{21}$$

$$W \equiv \rho_{22} - \rho_{11}$$

$$W - W_\infty = \rho_{22} - \rho_{11} - \frac{e^{-\frac{\hbar\omega_0}{kT}} - 1}{e^{-\frac{\hbar\omega_0}{kT}} + 1}$$

$$\begin{aligned}
\frac{dW}{dt} &= [-i(\eta\rho_{21} - \eta^*\rho_{12}) - \Gamma_2\rho_{22} + \Gamma_1\rho_{11}] - [i(\eta\rho_{21} - \eta^*\rho_{12}) - \Gamma_1\rho_{11} + \Gamma_2\rho_{22}] \\
&= -i\ 2(\eta\rho_{21} - \eta^*\rho_{12}) - 2\Gamma_2\rho_{22} + 2\Gamma_1\rho_{11} \\
&= -i\ 2(\eta\rho_{21} - \eta^*\rho_{12}) - \Gamma_2(e^{-\frac{\hbar\omega_0}{kT}} + 1)(W - W_\infty) \\
&= -i\ 2(\eta\rho_{21} - \eta^*\rho_{12}) - \gamma_1(W - W_\infty)
\end{aligned}$$

$$u \equiv \rho_{12} + \rho_{21}$$

$$\begin{aligned}
\frac{du}{dt} &= i\omega_0\rho_{12} + i\eta(\rho_{22} - \rho_{11}) - \gamma_2\rho_{12} - i\omega_0\rho_{21} - i\eta^*(\rho_{22} - \rho_{11}) - \gamma_2\rho_{21} \\
&= i(\eta - \eta^*)W - \gamma_2 u - \omega_0 v
\end{aligned}$$

$$v = i(\rho_{21} - \rho_{12})$$

$$\begin{aligned}
\frac{dv}{dt} &= i[-i\omega_0\rho_{21} - i\eta^*(\rho_{22} - \rho_{11}) - \gamma_2\rho_{21} - i\omega_0\rho_{12} - i\eta(\rho_{22} - \rho_{11}) + \gamma_2\rho_{12}] \\
&= \omega_0\rho_{21} + \eta^*(\rho_{22} - \rho_{11}) - i\gamma_2\rho_{21} + \omega_0\rho_{12} + \eta(\rho_{22} - \rho_{11}) + i\gamma_2\rho_{12} \\
&= (\eta^* + \eta)W - \gamma_2 v + \omega_0 u
\end{aligned}$$

緩和がなければ

$$|u|^2+|v|^2+|w|^2=|\rho_{12}+\rho_{21}|^2+|\rho_{21}-\rho_{12}|^2+|\rho_{22}-\rho_{11}|^2$$
$$=2|\rho_{12}|^2+2|\rho_{21}|^2+|\rho_{22}|^2+|\rho_{11}|^2-2\rho_{22}\rho_{11}$$
$$=4|C_1|^2|C_2|^2+|C_1|^4+|C_2|^4-2|C_1|^2|C_2|^2=(|C_1|^2+|C_2|^2)^2=1$$

$u+iv=2\rho_{12}\quad u-iv=2\rho_{21}$

$$\frac{dW}{dt}=-i[\eta(u-iv)-\eta^*(u+iv)]-\gamma_1(W-W_\infty)$$
$$=-i[(\eta-\eta^*)u-i(\eta+\eta^*)v]-\gamma_1(W-W_\infty)$$

$$\frac{du}{dt}=i(\eta-\eta^*)W-\gamma_2 u-\omega_0 v$$

$$\frac{dv}{dt}=(\eta^*+\eta)W-\gamma_2 v+\omega_0 u$$

$$\frac{d(u+iv)}{dt}=i(\eta-\eta^*)W-\gamma_2 u-\omega_0 v+i(\eta+\eta^*)W-i\gamma_2 v+i\omega_0 u$$
$$=i2\eta W-\gamma_2(u+iv)+i\omega_0(u+iv)$$
$$=i\xi(e^{i\omega t}+e^{-i\omega t})W-\gamma_2(u+iv)+i\omega_0(u+iv)$$

$$\frac{d(u-iv)}{dt}=i(\eta-\eta^*)W-\gamma_2 u-\omega_0 v-i(\eta+\eta^*)W+i\gamma_2 v-i\omega_0 u$$
$$=-i2\eta^*W-\gamma_2(u-iv)-i\omega_0(u-iv)$$
$$=-i\xi^*(e^{i\omega t}+e^{-i\omega t})W-\gamma_2(u-iv)-i\omega_0(u-iv)$$

$$(u+iv)e^{-i\omega t}=U+iV$$
$$(u-iv)e^{i\omega t}=U-iV$$

$$\frac{dW}{dt}=-i[\eta(u-iv)-\eta^*(u+iv)]-\gamma_1(W-W_\infty)$$
$$=-i\left[\xi\frac{e^{i\omega t}+e^{-i\omega t}}{2}(U-iV)e^{-i\omega t}-\xi^*\frac{e^{-i\omega t}+e^{i\omega t}}{2}(U+iV)e^{i\omega t}\right]-\gamma_1(W-W_\infty)$$
$$=-i\left[\frac{\xi}{2}(U-iV)(1+e^{-i2\omega t})-\frac{\xi^*}{2}(U+iV)(1+e^{i2\omega t})\right]-\gamma_1(W-W_\infty)$$

$$\frac{d(U+iV)}{dt}=\frac{d(u+iv)}{dt}e^{-i\omega t}-i\omega(U+iV)$$
$$=i\xi(1+e^{-i2\omega t})W-\gamma_2(u+iv)e^{-i\omega t}+i\omega_0(u+iv)e^{-i\omega t}-i\omega(U+iV)$$
$$=i\xi(1+e^{-i2\omega t})W-\gamma_2(U+iV)+i(\omega_0-\omega)(U+iV)$$

回転波近似

$$\frac{dW}{dt}\cong-i\left[\frac{\xi}{2}(U-iV)-\frac{\xi^*}{2}(U+iV)\right]-\gamma_1(W-W_\infty)$$

$$\frac{d(U+iV)}{dt}\cong i\xi W-\gamma_2(U+iV)+i(\omega_0-\omega)(U+iV)$$

$$\boxed{\begin{aligned}\frac{dW}{dt}&\cong(\mathrm{Im}\,\xi)U-(\mathrm{Re}\,\xi)V-\gamma_1(W-W_\infty)\\ \frac{dU}{dt}&\cong-(\mathrm{Im}\,\xi)W-\gamma_2 U-(\omega_0-\omega)V\\ \frac{dV}{dt}&\cong(\mathrm{Re}\,\xi)W-\gamma_2 V+(\omega_0-\omega)U\end{aligned}}$$

Bloch**方程式**

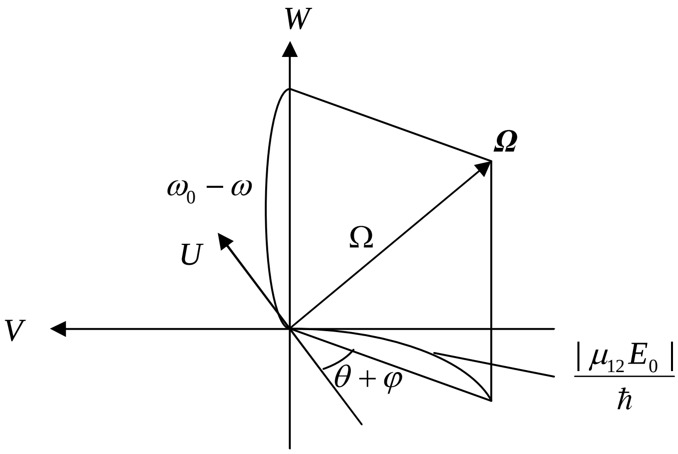
Bloch**方程式**

$$\frac{dU}{dt} = -(\mathrm{Im}\,\xi)W - (\omega_0 - \omega)V - \gamma_2 U$$

$$\frac{dV}{dt} = (\mathrm{Re}\,\xi)W + (\omega_0 - \omega)U - \gamma_2 V$$

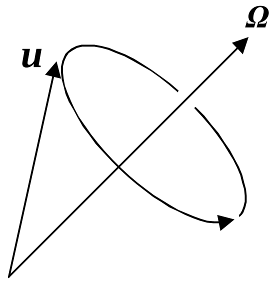
$$\frac{dW}{dt} = (\mathrm{Im}\,\xi)U - (\mathrm{Re}\,\xi)V - \gamma_1 (W - W_\infty)$$

$$\boldsymbol{u} = \begin{pmatrix} U \\ V \\ W \end{pmatrix} \quad \boldsymbol{\Omega} = \begin{pmatrix} -\mathrm{Re}\,\xi \\ -\mathrm{Im}\,\xi \\ \omega_0 - \omega \end{pmatrix}$$



$$\frac{d\boldsymbol{u}}{dt} = \boldsymbol{\Omega} \times \boldsymbol{u} + (\text{緩和項})$$

$$\boldsymbol{\Omega} \times \boldsymbol{u} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ -\mathrm{Re}\,\xi & -\mathrm{Im}\,\xi & \omega_0 - \omega \\ U & V & W \end{vmatrix}$$



$$E_0 = |E_0| e^{i\theta}$$

光電場$E = |E_0| \cos(\omega t + \theta)$

$$\xi = \frac{\mu_{12} E_0}{\hbar}$$

$$\mu_{12} = |\mu_{12}| e^{i\varphi}$$

$\theta + \varphi = 0$**のとき、**$\mathrm{Re}\,\xi = \xi \quad \mathrm{Im}\,\xi = 0$

定常解

$$-(\omega_0 - \omega)V - \gamma_2 U = 0$$

$$\xi W + (\omega_0 - \omega)U - \gamma_2 V = 0$$

$$-\xi V - \gamma_1 (W - W_\infty) = 0$$

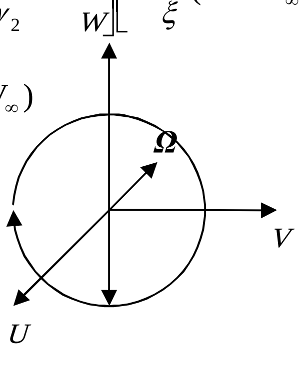
$$V = -\frac{\gamma_1}{\xi}(W - W_\infty)$$

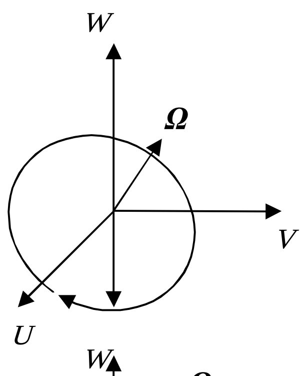
$$U = -\frac{(\omega_0 - \omega)}{\gamma_2} V$$

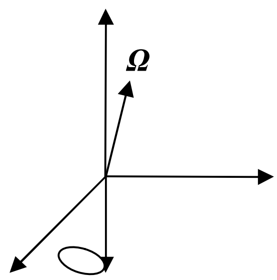
$$W = \frac{(\omega_0 - \omega)^2 + {\gamma_2}^2}{\xi \gamma_2} V = \left[\frac{(\omega_0 - \omega)^2 + {\gamma_2}^2}{\xi \gamma_2}\right]\left[-\frac{\gamma_1}{\xi}(W - W_\infty)\right]$$

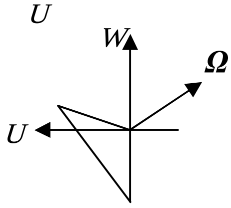
$$= -\frac{\gamma_1}{\xi}\left[\frac{(\omega_0 - \omega)^2 + {\gamma_2}^2}{\xi \gamma_2}\right](W - W_\infty)$$

$$W = \frac{\frac{\gamma_1}{\xi}\left[\frac{(\omega_0 - \omega)^2 + {\gamma_2}^2}{\xi \gamma_2}\right]}{1 + \frac{\gamma_1}{\xi}\left[\frac{(\omega_0 - \omega)^2 + {\gamma_2}^2}{\xi \gamma_2}\right]} W_\infty$$







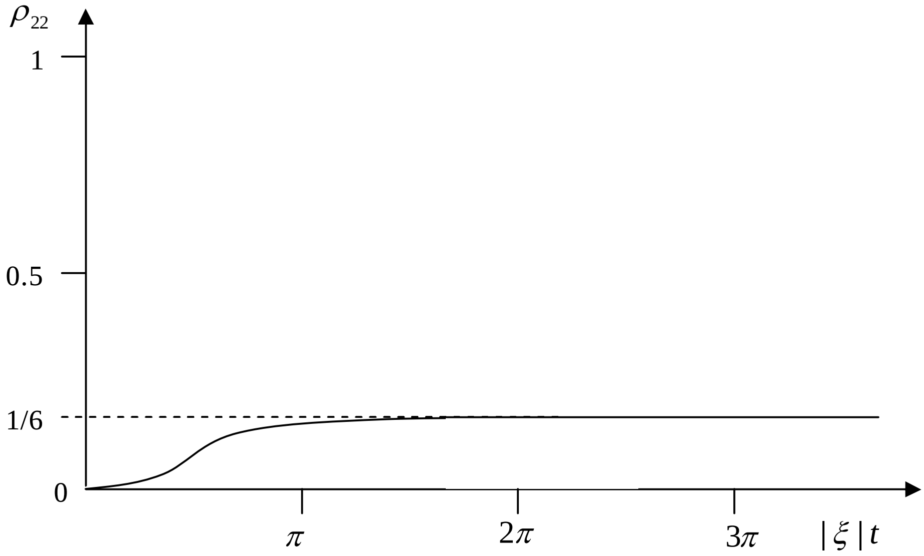


緩和があるとき t が十分大きいところで

Bloch **ベクトルは静止したベクトルになる**

（回転系で静止＝ωで回転している）

$\omega = \omega_0 \quad W_\infty = -1 \quad \gamma_1 = 2\gamma_2 \quad \dfrac{\gamma_2}{\xi} = 1$ **のとき**



$$W = \frac{-\dfrac{2\gamma_2{}^2}{\xi^2}}{1+\dfrac{2\gamma_2{}^2}{\xi^2}} = -\frac{2}{3}$$

$$W = \rho_{22} - \rho_{11} = -\frac{2}{3} \quad \rho_{22} + \rho_{11} = 1$$

$$\rho_{22} = \frac{1}{6} \quad \rho_{11} = \frac{5}{6}$$

$$V = -\frac{2\gamma_2}{\xi}(W+1) = -\frac{2}{3}$$

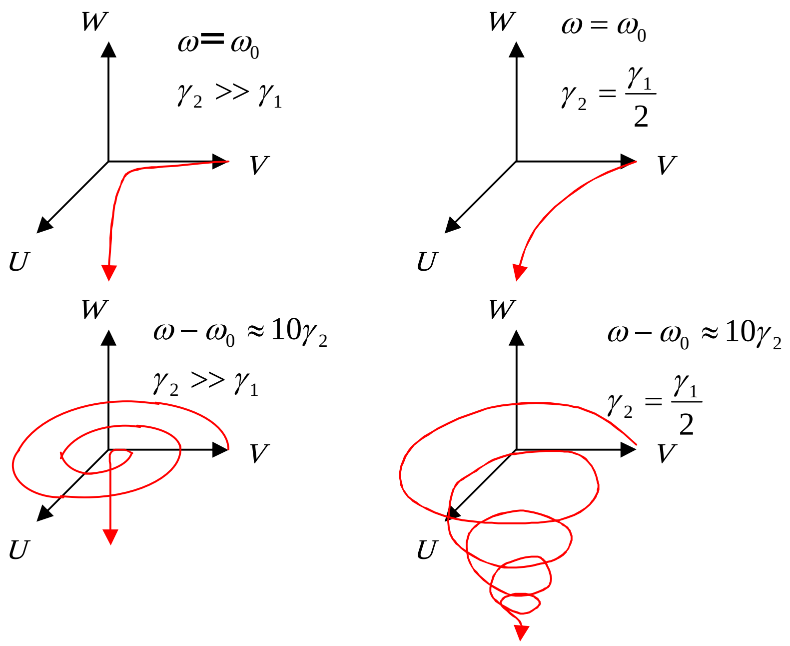
$$U = 0$$

$$U + iV = (u+iv)e^{-i\omega t}$$

$$2\rho_{12} = u + iv = -\frac{2}{3} i e^{i\omega t}$$

$$\rho_{12} = -\frac{1}{3} i e^{i\omega t}$$

$t=0$でBloch Vector $\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ **外場offになったときの**Bloch Vector**の変化**



$$\gamma_2 = (\frac{\Gamma_1}{2} + \frac{\Gamma_2}{2}) + \gamma_p$$

Γ_1, Γ_2 : **準位**1,2**のエネルギー緩和率**(energy or population relaxation rate), **縦緩和**

γ_2 : 1 – 2**間の分極の位相緩和率**(phase relaxation rate, polarization decay rate), **横緩和**

γ_p : pure dephasing rate(**エネルギー緩和しないで位相のみ緩和する**)

$$\begin{cases} \dfrac{dC_1}{dt} = -i\omega_1 C_1 + i\eta C_2 - \dfrac{\Gamma_1}{2} C_1 \\ \dfrac{dC_2}{dt} = -i\omega_2 C_2 + i\eta^* C_1 - \dfrac{\Gamma_2}{2} C_2 \end{cases}$$

$$\begin{cases} \dfrac{d\rho_{22}}{dt} = \dfrac{dC_2}{dt} C_2{}^* + C_2 \dfrac{dC_2{}^*}{dt} = i(\eta^* \rho_{12} - \eta\rho_{21}) - \Gamma_2 \rho_{22} \\ \dfrac{d\rho_{11}}{dt} = i(\eta\rho_{21} - \eta^* \rho_{12}) - \Gamma_1 \rho_{11} \\ \dfrac{d\rho_{12}}{dt} = \dfrac{dC_1}{dt} C_2{}^* + C_1 \dfrac{dC_2{}^*}{dt} = i(\omega_2 - \omega_1)\rho_{12} + i\eta(\rho_{22} - \rho_{11}) - (\dfrac{\Gamma_1}{2} + \dfrac{\Gamma_2}{2})\rho_{12} \end{cases}$$

$$\downarrow$$

$$\gamma_2 = \frac{\Gamma_1}{2} + \frac{\Gamma_2}{2} + \gamma_p$$

準位1**が基底状態**$\Gamma_1 = 0$**のとき** $\gamma_2 = \dfrac{\Gamma_2}{2} + \gamma_p$ $\Gamma_2 = \gamma_1$**とおく** $\gamma_2 = \dfrac{\gamma_1}{2} + \gamma_p$ $\gamma_p = 0$**のときは**γ_2**は**γ_1**のみで決まる**

分極$P(t) = P_0 e^{i\omega_0 t - \gamma_2 t}$

population $\propto |P(t)|^2 = P_0{}^2 e^{-2\gamma_2 t} = P_0{}^2 e^{-\gamma_1 t}$

吸収線幅がエネルギー緩和時間でなく位相緩和時間で決まる理由

$P(\omega) = \varepsilon_0 \chi(\omega) E(\omega)$

$P(t) = \varepsilon_0 \int_0^\infty d\tau \chi(\tau) E(t-\tau) = \varepsilon_0 \int_{-\infty}^t d\tau \chi(t-\tau) E(\tau)$

$t > 0$**のとき**

$$\chi(\omega) \propto \frac{1}{\omega_0{}^2 - \omega^2 - i\omega\Gamma_0} \qquad \gamma_2 = \Gamma_0 / 2$$

$$\chi(\omega) \propto \frac{\omega_0{}^2 - \omega^2}{(\omega_0{}^2 - \omega^2)^2 + \omega^2\,\Gamma_0{}^2} + i\frac{\omega\,\Gamma_0}{(\omega_0{}^2 - \omega^2)^2 + \omega^2\,\Gamma_0{}^2}$$

ω〜ω_0**付近の近似式**

$$\chi(\omega) \propto \frac{(\omega_0+\omega)(\omega_0-\omega)}{(\omega_0+\omega)^2(\omega_0-\omega)^2 + \omega^2\,\Gamma_0{}^2} + i\frac{\omega\,\Gamma_0}{(\omega_0+\omega)^2(\omega_0-\omega)^2 + \omega^2\,\Gamma_0{}^2}$$

$$\sim \frac{2\omega_0(\omega_0-\omega)}{4\omega_0{}^2(\omega_0-\omega)^2 + \omega_0{}^2\,\Gamma_0{}^2} + i\frac{\omega_0\,\Gamma_0}{4\omega_0{}^2(\omega_0-\omega)^2 + \omega_0{}^2\,\Gamma_0{}^2}$$

$$= \frac{(\omega_0-\omega)}{2\omega_0(\omega_0-\omega)^2 + 2\omega_0\,(\Gamma_0/2)^2} + i\frac{\Gamma_0/2}{2\omega_0(\omega_0-\omega)^2 + 2\omega_0\,(\Gamma_0/2)^2}$$

$$\propto \frac{(\omega_0-\omega)}{(\omega_0-\omega)^2 + (\Gamma_0/2)^2} + i\frac{\Gamma_0/2}{(\omega_0-\omega)^2 + (\Gamma_0/2)^2}$$ **虚部が吸収スペクトルに比例　半値半幅**$\Gamma_0/2$

$$\chi(t) = \frac{1}{2\pi}\int \chi(\omega) e^{-i\omega t} d\omega \propto \int \frac{e^{-i\omega t}}{\omega_0{}^2 - \omega^2 - i\omega\Gamma_0} d\omega$$

$$\chi(t) = \frac{1}{2\pi}\int \chi(\omega) e^{-i\omega t} d\omega = \frac{1}{2\pi}\frac{1}{\varepsilon_0}\frac{N_0}{V}\frac{q^2}{m}\int \frac{e^{-i\omega t}}{\omega_0{}^2 - \omega^2 - i\omega\Gamma_0} d\omega = \frac{1}{\varepsilon_0}\frac{N_0}{V}\frac{q^2}{m}\frac{e^{-\frac{\Gamma_0}{2}t}}{\sqrt{\omega_0{}^2 - \frac{\Gamma_0{}^2}{4}}}\sin\left(\sqrt{\omega_0{}^2 - \frac{\Gamma_0{}^2}{4}}\,t\right)$$

$t < 0$　**のときは　複素**ω**平面の上半平面での複素積分　より**　$\chi(t) = 0$

広帯域入射光$E(t) = E_0\delta(t)$**に対する応答**$\rightarrow$**フーリエ変換**　　flat**なスペクトル**

$P(t) = \varepsilon_0 \int_0^\infty d\tau \chi(\tau) E(t-\tau) = \varepsilon_0 \int_0^\infty d\tau \chi(\tau) E_0 \delta(t-\tau) = \varepsilon_0 E_0 \chi(t)$　**周波数**$\omega = \sqrt{\omega_0{}^2 - \frac{\Gamma_0{}^2}{4}}$**の分極が**$e^{-\gamma_2 t}$**で減衰**

この分極による２次波がちょうど入射光を逆位相で打ち消す$\rightarrow$ **透過の減少＝吸収**

$\omega_0 >> \Gamma_0/2$**なので**$\omega \approx \omega_0$

減衰分極振動$\chi(t) \propto \mathrm{Re}[i\theta(t)e^{-i\omega_0 t - \gamma_2 t}]$**のフーリエ変換**

$$i\int_{-\infty}^\infty \theta(t) e^{-i\omega_0 t - \gamma_2 t} e^{i\omega t} dt = i\int_0^\infty e^{-i(\omega_0-\omega)t - \gamma_2 t} dt = i\frac{-1}{-i(\omega_0-\omega) - \gamma_2} = \frac{1}{(\omega_0-\omega) - i\gamma_2}$$

の虚部は$\gamma_2 (= \Gamma_0/2)$**の半値半幅を持つ。つまり吸収線の幅は位相緩和時間**

(polarization decay time)**で決まる。**