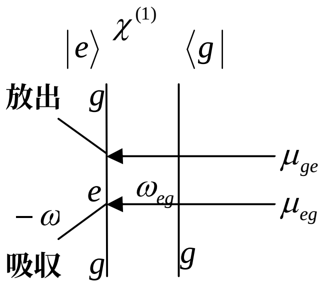
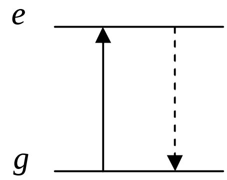
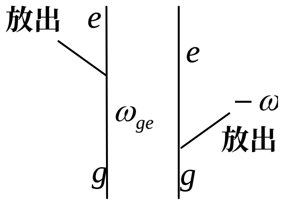
$$\omega_0 = \omega_{eg} = \omega_e - \omega_g = \omega_{21}$$

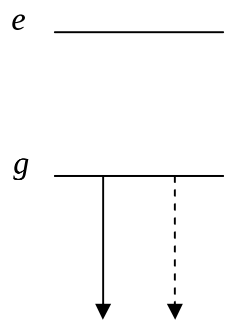




$$\chi^{(1)}(\omega;-\omega) \propto \frac{1}{\omega_0 - \omega - i\gamma_2}$$

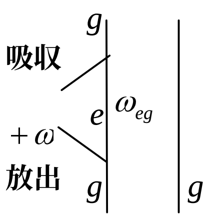
共鳴項 μ_{ge} $\rho_{eg}(+\omega)$ $e^{-i\omega t}$

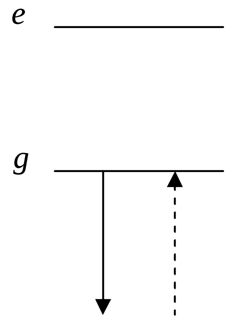




$$\chi^{(1)}(\omega;-\omega) \propto \frac{-1}{-\omega_0 - \omega - i\gamma_2} = \frac{1}{\omega_0 + \omega + i\gamma_2}$$

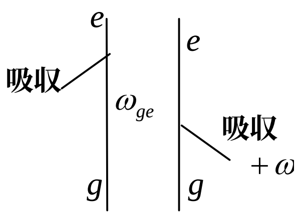
非共鳴項 μ_{eg} $\rho_{ge}(+\omega)$ $e^{-i\omega t}$

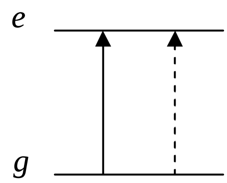




$$\chi^{(1)}(-\omega;\omega) \propto \frac{1}{\omega_0 + \omega - i\gamma_2}$$

非共鳴項 μ_{ge} $\rho_{eg}(-\omega)$ $e^{i\omega t}$





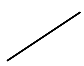
$$\chi^{(1)}(-\omega;\omega) \propto \frac{-1}{-\omega_0 + \omega - i\gamma_2} = \frac{1}{\omega_0 - \omega + i\gamma_2}$$

共鳴項 μ_{eg} $\rho_{ge}(-\omega)$ $e^{i\omega t}$

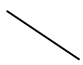
$$P^{(1)}(t) = \varepsilon_0 \frac{E_0}{2}[\chi^{(1)}(\omega)e^{-i\omega t} + \chi^{(1)}(-\omega)e^{i\omega t}] \qquad E(t) = \frac{E_0}{2}(e^{-i\omega t} + e^{i\omega t})$$

$$\chi^{(1)}(\omega) = \chi^{(1)}(\omega;-\omega) = \frac{N}{V}\frac{|\mu_{12}|^2}{\varepsilon_0 \hbar}\left(\frac{1}{\omega_0 - \omega - i\gamma_2} + \frac{1}{\omega_0 + \omega + i\gamma_2}\right)$$

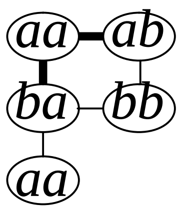
$$\chi^{(1)}(-\omega) = \chi^{(1)}(-\omega;\omega) = \frac{N}{V}\frac{|\mu_{12}|^2}{\varepsilon_0 \hbar}\left(\frac{1}{\omega_0 + \omega - i\gamma_2} + \frac{1}{\omega_0 - \omega + i\gamma_2}\right)$$

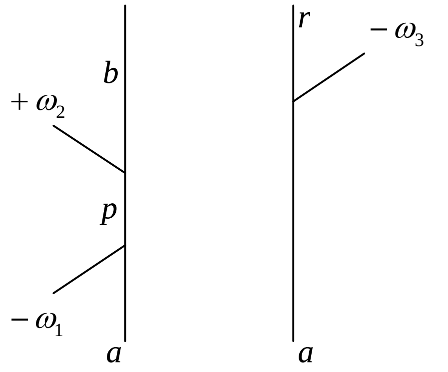


右上がり $-\omega$



右下がり $+\omega$





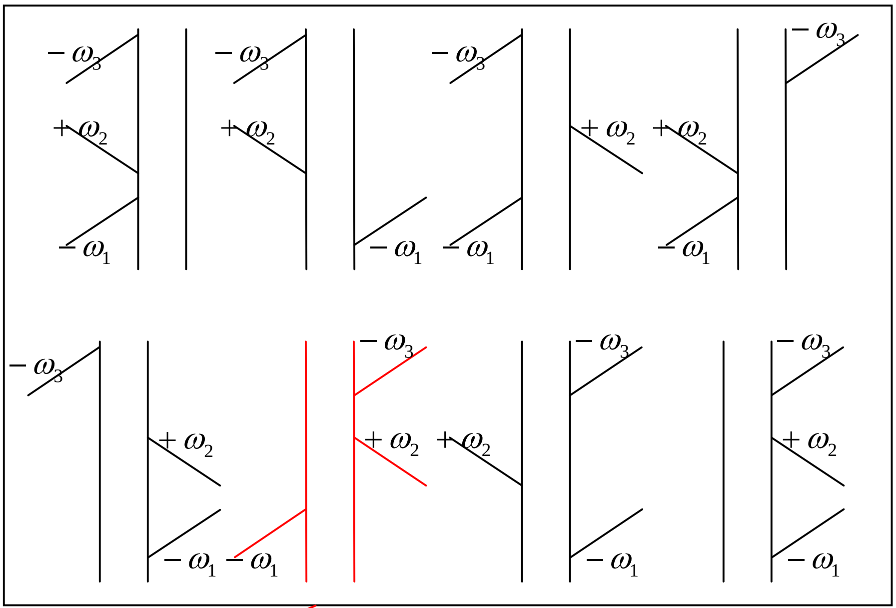
$arbp$ 2準位系 gかe

$a = g$

$r, b, p = e,e,e$
g,e,e
e,g,e
e,e,g
g,g,e
g,e,g
e,g,g
g,g,g

反転対称性がない場合

$\mu_{gg}, \mu_{ge}, \mu_{eg}, \mu_{ee} \neq 0$

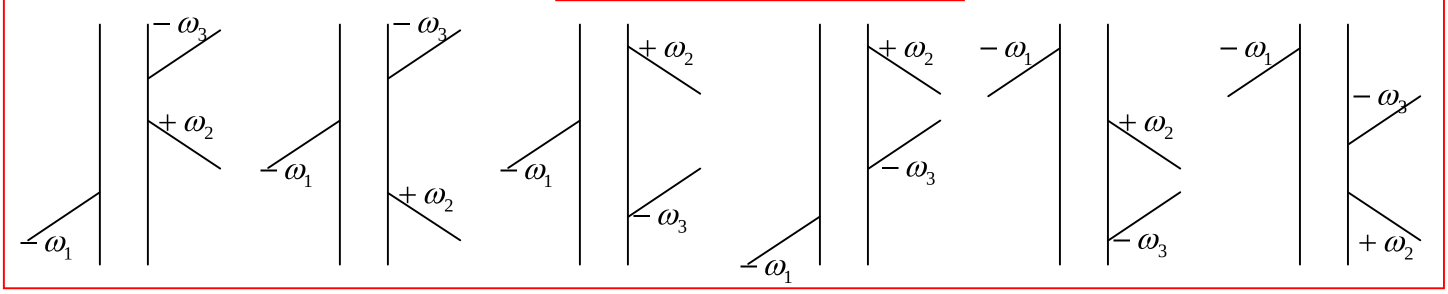


左右どちらにω_jがくるか

$2\times2\times2=8$通り

計$8\times6=48$通り

$\omega_1, \omega_2, \omega_3$の順列　$3!=6$通り



上のダイアグラムに対応する$\chi^{(3)}$

$$\chi^{(3)}_{klmn}(\omega_3-\omega_2+\omega_1;-\omega_3,\omega_2,-\omega_1)$$

$$=\frac{N}{V}\frac{1}{\varepsilon_0\hbar^3}\sum_{arbp}\mu^l_{ar}\mu^k_{rb}\mu^m_{bp}\mu^n_{pa}\frac{-1}{\omega_{br}-(\omega_3-\omega_2+\omega_1)-i\gamma_{br}}\frac{1}{\omega_{ba}-(-\omega_2+\omega_1)-i\gamma_{ba}}\frac{1}{\omega_{pa}-\omega_1-i\gamma_{pa}}\rho^{(0)}_{aa}$$

$$P^{(3)}_k(\omega_3-\omega_2+\omega_1)=\varepsilon_0\chi^{(3)}_{klmn}(\omega_3-\omega_2+\omega_1;-\omega_3,\omega_2,-\omega_1)\left(\frac{E_l(\omega_3)}{2}e^{-i\omega_3 t}\right)\left(\frac{E_m(-\omega_2)}{2}e^{i\omega_2 t}\right)\left(\frac{E_n(\omega_1)}{2}e^{-i\omega_1 t}\right)$$

2準位系で反転対称性がある場合

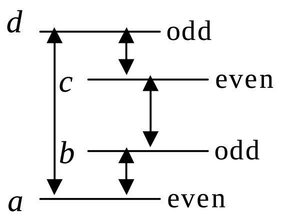
$\mu_{pa}, \mu_{bp}, \mu_{rb}, \mu_{ar} = \mu_{ge}$ or μ_{eg}　$a=g$から出発すれば3回の遷移は$g\to e\to g\to e$のみ　1通り

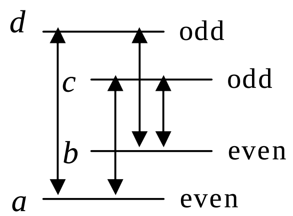
4準位系で反転対称性がある場合

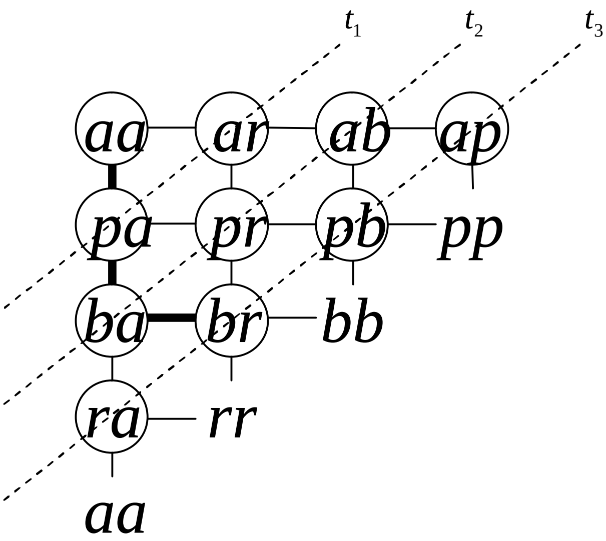
3回の遷移ごとに2つの可能性

$2\times2\times2=8$通り

計$48\times8=384$通り







期待値$P=\frac{N}{V}\mathrm{Tr}(\rho\mu)$

$\rho_{br}\mu_{rb}$ or $\mu_{rb}\rho_{br}$

$$\begin{aligned}\langle\psi|A|\psi\rangle &= \langle C_1\phi_1+C_2\phi_2|A|C_1\phi_1+C_2\phi_2\rangle\\ &= C_1{}^*C_1A_{11}+C_2{}^*C_1A_{21}+C_1{}^*C_2A_{12}+C_2{}^*C_2A_{22}\\ &= \rho_{11}A_{11}+\rho_{12}A_{21}+\rho_{21}A_{12}+\rho_{22}A_{22}\\ &= \mathrm{Tr}(\rho A)=\mathrm{Tr}(A\rho)\end{aligned}$$

$$\because\ \rho A=\begin{pmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{pmatrix}\begin{pmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{pmatrix}=\begin{pmatrix}\boxed{\rho_{11}A_{11}+\rho_{12}A_{21}} & \rho_{11}A_{12}+\rho_{12}A_{22}\\ \rho_{21}A_{11}+\rho_{22}A_{21} & \boxed{\rho_{21}A_{12}+\rho_{22}A_{22}}\end{pmatrix}$$

$$\because A\rho=\begin{pmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{pmatrix}\begin{pmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{pmatrix}=\begin{pmatrix}\boxed{A_{11}\rho_{11}+A_{12}\rho_{21}} & A_{11}\rho_{12}+A_{12}\rho_{22}\\ A_{21}\rho_{11}+A_{22}\rho_{21} & \boxed{A_{21}\rho_{12}+A_{22}\rho_{22}}\end{pmatrix}$$

巨視的分極

$$P(t)=\frac{N}{V}(\rho_{21}\mu_{12}+\rho_{12}\mu_{21})=\frac{N}{V}(\mu_{12}\rho_{21}+\mu_{21}\rho_{12})$$