Radiation Reaction pp.469-472

$$Rc-\boldsymbol{R}\cdot\boldsymbol{v}=\boldsymbol{R}\cdot\boldsymbol{u}\quad \text{with } \boldsymbol{u}\equiv c\hat{\boldsymbol{R}}-\boldsymbol{v}$$

$$\boldsymbol{E}(\boldsymbol{r},t)=\frac{q}{4\pi\varepsilon_0}\frac{R}{(\boldsymbol{R}\cdot\boldsymbol{u})^3}[(c^2-v^2)\boldsymbol{u}+\boldsymbol{R}\times(\boldsymbol{u}\times\boldsymbol{a})]$$

$$=\frac{q}{4\pi\varepsilon_0}\frac{R}{(\boldsymbol{R}\cdot\boldsymbol{u})^3}[(c^2-v^2)\boldsymbol{u}+(\boldsymbol{R}\cdot\boldsymbol{a})\boldsymbol{u}-(\boldsymbol{R}\cdot\boldsymbol{u})\boldsymbol{a}]\qquad (10.65)\quad v=0\quad\rightarrow\quad (11.85)$$

$\boldsymbol{a}$: x**軸方向加速度**

(11.88) $\boldsymbol{u}=c\hat{\boldsymbol{R}}\quad |\boldsymbol{u}|=c\quad u_x=c\dfrac{l}{R}$

(11.89)
$$E_{lx}=\frac{q/2}{4\pi\varepsilon_0}\frac{R}{(cR)^3}[(c^2+(l\hat{\boldsymbol{x}}+d\hat{\boldsymbol{y}})\cdot\boldsymbol{a})u_x-(cR)a]$$
$$=\frac{q/2}{4\pi\varepsilon_0}\frac{R}{(cR)^3}[(c^2+la)c\frac{l}{R}-(cR)a]$$
$$=\frac{q/2}{4\pi\varepsilon_0}\frac{1}{c^2R^3}[(c^2+la)l-(R^2)a]$$
$$=\frac{q/2}{4\pi\varepsilon_0}\frac{1}{c^2(l^2+d^2)^{3/2}}[(c^2+la)l-(l^2+d^2)a]$$
$$=\frac{q}{8\pi\varepsilon_0c^2}\frac{lc^2-ad^2}{(l^2+d^2)^{3/2}}$$

$\dot{x}(t_r)=0$ instantaneously at rest $\rightarrow$ (11.91)

$$d=\sqrt{(cT)^2-l^2}=\sqrt{(cT)^2-(\frac{1}{2}aT^2+\frac{1}{6}\dot{a}T^3+\cdots)^2}=cT\sqrt{1-(\frac{1}{2c}aT+\frac{1}{6c}\dot{a}T^2+\cdots)^2}$$

(11.94)
$$l=\frac{1}{2}aT^2+\frac{1}{6}\dot{a}T^3+\cdots=\frac{1}{2}a\left(\frac{1}{c}d+\frac{a^2}{8c^5}d^3+()d^4+\cdots\right)^2+\frac{1}{6}\dot{a}\left(\frac{1}{c}d+\frac{a^2}{8c^5}d^3+()d^4+\cdots\right)^3+\cdots$$
$$=\frac{1}{2}a\left(\frac{1}{c^2}d^2+2\frac{1}{c}d\,\frac{a^2}{8c^5}d^3+\cdots\right)+\frac{1}{6}\dot{a}\left(\frac{1}{c^3}d^3+3(\frac{1}{c}d)^2\,\frac{a^2}{8c^5}d^3+\cdots\right)+\cdots$$
$$=\frac{1}{2}a\left(\frac{1}{c^2}d^2\right)+\frac{1}{6}\dot{a}\left(\frac{1}{c^3}d^3\right)+()d^4+\cdots$$

(11.95)
$$lc^2-ad^2=-\frac{a}{2}d^2+\frac{\dot{a}}{6c}d^3+()d^4+\cdots$$
$$l^2+d^2=(\frac{a^2}{4c^4}d^4+\cdots)+d^2=d^2+()d^4+\cdots$$
$$\frac{lc^2-ad^2}{(l^2+d^2)^{3/2}}=\frac{-\frac{a}{2}d^2+\frac{\dot{a}}{6c}d^3+()d^4+\cdots}{d^3}=-\frac{a}{2d}+\frac{\dot{a}}{6c}+()d+\cdots$$

(11.96)
$$\boldsymbol{F}_{\text{self}}=\frac{q^2}{4\pi\varepsilon_0}\left[-\frac{a(t_r)}{4c^2d}+\frac{\dot{a}(t_r)}{12c^3}+()d+\cdots\right]\hat{\boldsymbol{x}}=\frac{q^2}{4\pi\varepsilon_0}\left[-\frac{a(t)-\dot{a}(t)d/c}{4c^2d}+\frac{\dot{a}(t)}{12c^3}+()d+\cdots\right]\hat{\boldsymbol{x}}$$
$$=\frac{q^2}{4\pi\varepsilon_0}\left[-\frac{a(t)}{4c^2d}+\frac{\dot{a}(t)}{4c^3}+\frac{\dot{a}(t)}{12c^3}+()d+\cdots\right]\hat{\boldsymbol{x}}=\frac{q^2}{4\pi\varepsilon_0}\left[-\frac{a(t)}{4c^2d}+\frac{\dot{a}(t)}{3c^3}+()d+\cdots\right]\hat{\boldsymbol{x}}$$

(11.97)
$$\boldsymbol{F}_{\text{self}}=2m_0a\qquad \frac{q^2}{4\pi\varepsilon_0}\left[-\frac{a(t)}{4c^2d}+\frac{\dot{a}(t)}{3c^3}+()d+\cdots\right]=2m_0a$$
$$\left(2m_0+\frac{q^2}{4\pi\varepsilon_0}\frac{1}{4c^2d}\right)a=\frac{q^2}{4\pi\varepsilon_0}\left[\frac{\dot{a}(t)}{3c^3}+()d+\cdots\right]\qquad m=2m_0+\frac{1}{4\pi\varepsilon_0}\frac{q^2}{4dc^2}=2m_0+m'$$

自己場の衣を着た質量

$$E=\frac{1}{4\pi\varepsilon_0}\frac{(q/2)^2}{d}=m'c^2$$

繰り込まれた質量

radiation reaction の大きさ　p.467

(11.80)　$F_{\mathrm{rad}}=\dfrac{\mu_0 q^2}{6\pi c}\dot{a}=\tau m\dot{a}=\tau\dot{F}$　$\tau=6\times10^{-24}\mathrm{s}$　for electron　(11.82)

$a=a_0\sin\omega t$　$\dot{a}=\omega a_0\cos\omega t$

$F_{\mathrm{rad}}=\tau\omega m a_0\cos\omega t$　$\omega=2\pi f<<\dfrac{1}{\tau}$　$f<<\dfrac{1}{2\pi\tau}=3\times10^{22}\,\mathrm{Hz}$

加速度の変化率が$3\times10^{22}\,\mathrm{Hz}$**より十分小さければ**$F_{\mathrm{rad}}$**は無視できる**