11.1.2 Electric Dipole Radiation

$$R_{\pm} = \sqrt{r^2 \mp rd\cos\theta + (d/2)^2} = r\left[1 \mp \frac{d}{r}\cos\theta + \frac{1}{r^2}\left(\frac{d}{2}\right)^2\right]^{\frac{1}{2}} \qquad (11.6)$$

approx.1 $\quad d << r$

$$R_{\pm} \cong r\left(1 \mp \frac{d}{2r}\cos\theta\right)$$

approx.2 $\quad d << \dfrac{c}{\omega} = \dfrac{\lambda}{2\pi} = \dfrac{1}{k}$

$$\cos[\omega(t - R_{\pm}/c)] \cong \cos[\omega(t - r/c)]\cos\left(\pi\frac{d}{\lambda}\cos\theta\right) \mp \sin[\omega(t - r/c)]\sin\left(\pi\frac{d}{\lambda}\cos\theta\right)$$

$$\cos[\omega(t - R_{\pm}/c)] \cong \cos[\omega(t - r/c)] \mp \pi\frac{d}{\lambda}\cos\theta\sin[\omega(t - r/c)] \qquad (11.11)$$

(11.5)

$$V(\boldsymbol{r},t) = \frac{1}{4\pi\varepsilon_0}\frac{q_0}{r}\left\{\left(1+\frac{d}{2r}\cos\theta\right)\cos[\omega(t - R_{+}/c)] - \left(1-\frac{d}{2r}\cos\theta\right)\cos[\omega(t - R_{-}/c)]\right\}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_0}{r}\left\{\begin{aligned}&\left(1+\frac{d}{2r}\cos\theta\right)\left[\cos[\omega(t - r/c)] - \pi\frac{d}{\lambda}\cos\theta\sin[\omega(t - r/c)]\right]\\ &\quad -\left(1-\frac{d}{2r}\cos\theta\right)\left[\cos[\omega(t - r/c)] + \pi\frac{d}{\lambda}\cos\theta\sin[\omega(t - r/c)]\right]\end{aligned}\right\}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_0}{r}\left\{\frac{d}{r}\cos\theta\cos[\omega(t - r/c)] - 2\pi\frac{d}{\lambda}\cos\theta\sin[\omega(t - r/c)]\right\}$$

$$= \frac{q_0 d\cos\theta}{4\pi\varepsilon_0 r}\left\{\frac{1}{r}\cos[\omega(t - r/c)] - \frac{2\pi}{\lambda}\sin[\omega(t - r/c)]\right\} \qquad (11.12)$$

approx.3 $\quad r >> \dfrac{c}{\omega} = \dfrac{\lambda}{2\pi} = \dfrac{1}{k}$

$$V(r,\theta,t) = \frac{q_0 d\cos\theta}{4\pi\varepsilon_0 r}\left\{-\frac{2\pi}{\lambda}\sin[\omega(t - r/c)]\right\} \qquad (11.14)$$

$$A(r,t)=\frac{\mu_0}{4\pi}\int_{-d/2}^{d/2}\frac{-q_0\omega\sin[\omega(t-R/c)]\hat{\boldsymbol{z}}}{R}dz \qquad (11.16)$$ **被積分関数はzに依存**

$$=\hat{\boldsymbol{z}}\frac{\mu_0}{4\pi}(-q_0\omega)\int_{-d/2}^{d/2}\frac{1}{r}(1+\frac{d}{2r}\cos\theta)\sin[\omega t-\frac{\omega}{c}r(1-\frac{d}{2r}\cos\theta)]dz$$

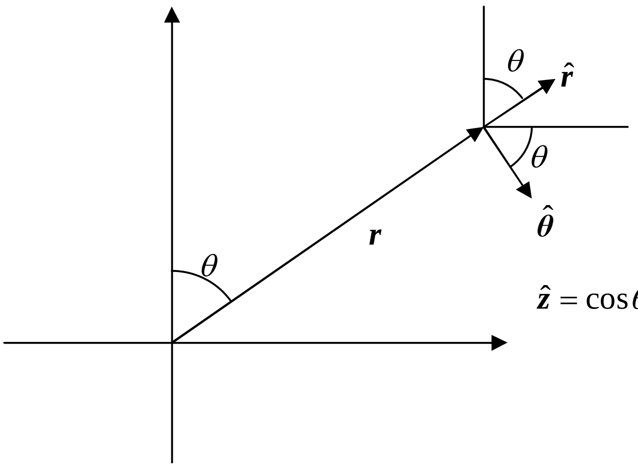
$$=\hat{\boldsymbol{z}}\frac{\mu_0}{4\pi}(-q_0\omega)\int_{-d/2}^{d/2}\frac{1}{r}(1+\frac{d}{2r}\cos\theta)\sin[\omega(t-\frac{r}{c})+\pi\frac{d}{\lambda}\cos\theta)]dz$$

$approx.1 \quad d << r \qquad\qquad approx.2 \quad d << \frac{c}{\omega}=\frac{\lambda}{2\pi}=\frac{1}{k}$

積分によってdの1次のオーダーになるので、被積分関数はdについて1次の項は落とす

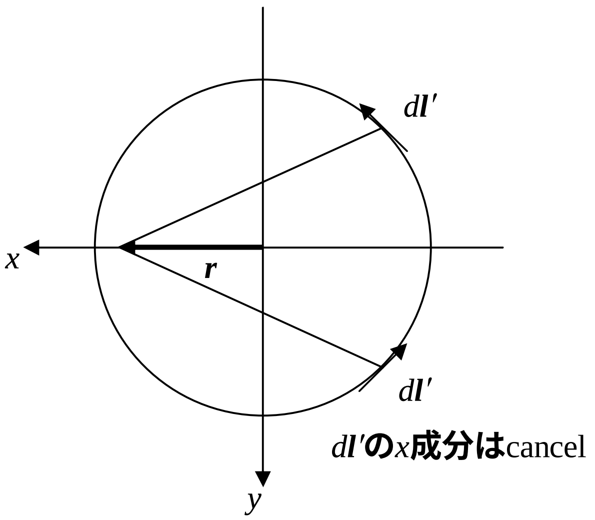
$$=\hat{\boldsymbol{z}}\frac{\mu_0}{4\pi}(-q_0\omega)\int_{-d/2}^{d/2}\frac{1}{r}\sin[\omega(t-\frac{r}{c})]dz$$ **被積分関数はzに依存しない**

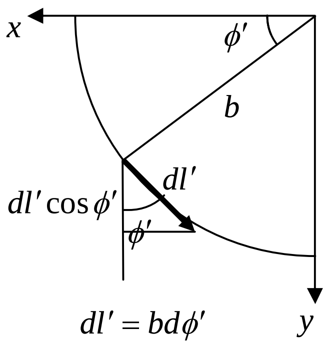
$$=-\frac{\mu_0 q_0 d\omega}{4\pi r}\sin[\omega(t-\frac{r}{c})]\hat{\boldsymbol{z}} \qquad (11.17)$$



$$\hat{\boldsymbol{z}}=\cos\theta\hat{\boldsymbol{r}}-\sin\theta\hat{\boldsymbol{\theta}}$$

11.1.3 Magnetic Dipole Radiation





(11.27)に(11.30)(11.32)を代入

$$\boldsymbol{A}(\boldsymbol{r},t)=\frac{\mu_0 I_0 b}{4\pi}\hat{\boldsymbol{y}}\int_0^{2\pi}\frac{\cos[\omega(t-R/c)]}{R}\cos\phi' d\phi'$$

$$=\frac{\mu_0 I_0 b}{4\pi}\hat{\boldsymbol{y}}\int_0^{2\pi}\frac{1}{r}\left(1+\frac{b}{r}\sin\theta\cos\phi'\right)\left\{\cos[\omega(t-r/c)]-\frac{\omega b}{c}\sin\theta\cos\phi'\sin[\omega(t-r/c)]\right\}\cos\phi' d\phi'$$

2nd - order term $\frac{b}{r}\cdot\frac{\omega b}{c}$ の項を無視

(11.35) $\rightarrow$ (11.37)

$$\boldsymbol{A}(r,\theta,t)=A_\phi(r,\theta,t)\hat{\boldsymbol{\varphi}}$$

$$\nabla\times\boldsymbol{A}=\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\phi\sin\theta)\hat{\boldsymbol{r}}+\frac{1}{r}\{-\frac{\partial}{\partial r}(rA_\phi)\}\hat{\boldsymbol{\theta}}$$

第1項 $\propto\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\{\frac{\sin^2\theta}{r}\sin[\omega(t-r/c)]\}=\frac{1}{r\sin\theta}\frac{2\sin\theta\cos\theta}{r}\sin[\omega(t-r/c)]=\frac{1}{r}\frac{2\cos\theta}{r}\sin[\omega(t-r/c)]$

第2項 $\propto\frac{1}{r}\{-\frac{\partial}{\partial r}(\sin\theta\sin[\omega(t-r/c)])\}=\frac{1}{r}\frac{\omega}{c}\sin\theta\cos[\omega(t-r/c)]$

approx.3 $\frac{1}{r}<<\frac{\omega}{c}$ 第1項を無視 第2項が(11.37)

11.1.4　Radiation from an Arbitrary Source

(11.50)　$\dot{\rho}(\frac{\hat{\boldsymbol{r}}\cdot\boldsymbol{r}'}{c}) >> \frac{1}{2}\ddot{\rho}(\frac{\hat{\boldsymbol{r}}\cdot\boldsymbol{r}'}{c})^2 \quad \dot{\rho} >> \ddot{\rho}(\frac{\hat{\boldsymbol{r}}\cdot\boldsymbol{r}'}{c}) \quad \frac{c}{\ddot{\rho}/\dot{\rho}} >> \hat{\boldsymbol{r}}\cdot\boldsymbol{r}'$

$V(\boldsymbol{r},t) \cong \frac{1}{4\pi\varepsilon_0}\int\frac{1}{r}(1+\frac{\hat{\boldsymbol{r}}\cdot\boldsymbol{r}'}{r})[\rho(\boldsymbol{r}',t_0)+\dot{\rho}(\boldsymbol{r}',t_0)(\frac{\hat{\boldsymbol{r}}\cdot\boldsymbol{r}'}{c})]d\tau'$　r'について1次まで残す

p.214　Prob.5.7　$\boldsymbol{p} = \int\boldsymbol{r}'\rho(\boldsymbol{r}',t)d\tau' \quad \nabla\cdot(x\boldsymbol{J}) = (\nabla x)\cdot\boldsymbol{J} + x\nabla\cdot\boldsymbol{J} = J_x + x(-\frac{\partial\rho}{\partial t})$

$$\dot{p}_x = \int x'\frac{\partial\rho}{\partial t}d\tau' = \int J_x d\tau' - \int\nabla\cdot(x'\boldsymbol{J})d\tau' = \int J_x d\tau' - \int(x'\boldsymbol{J})\cdot d\boldsymbol{S}' = \int J_x d\tau' \quad \therefore \dot{\boldsymbol{p}} = \int\boldsymbol{J}d\tau'$$

$$\nabla[\frac{\hat{\boldsymbol{r}}\cdot\dot{\boldsymbol{p}}(t_0)}{r}] = \nabla[\frac{\boldsymbol{r}\cdot\dot{\boldsymbol{p}}(t_0)}{r^2}] = \nabla[\frac{x\dot{p}_x+y\dot{p}_y+z\dot{p}_z}{x^2+y^2+z^2}]$$

$$= (\boldsymbol{i}\frac{\partial}{\partial x}+\boldsymbol{j}\frac{\partial}{\partial y}+\boldsymbol{k}\frac{\partial}{\partial z})[\frac{x\dot{p}_x+y\dot{p}_y+z\dot{p}_z}{x^2+y^2+z^2}]$$

$$= \boldsymbol{i}\left[\frac{\partial}{\partial x}(\frac{1}{x^2+y^2+z^2})(x\dot{p}_x+y\dot{p}_y+z\dot{p}_z) + (\frac{1}{x^2+y^2+z^2})\frac{\partial}{\partial x}(x\dot{p}_x+y\dot{p}_y+z\dot{p}_z)\right] + \boldsymbol{j}[\,] + \boldsymbol{k}[\,]$$

$\frac{\partial}{\partial x}(\frac{1}{x^2+y^2+z^2}) = -\frac{2x}{(x^2+y^2+z^2)^2} = -\frac{2x}{r^4}$　この項は無視できる

残るのは

$$\frac{1}{x^2+y^2+z^2}\left[\boldsymbol{i}\frac{\partial}{\partial x}(x\dot{p}_x+y\dot{p}_y+z\dot{p}_z) + \boldsymbol{j}\frac{\partial}{\partial y}(x\dot{p}_x+y\dot{p}_y+z\dot{p}_z) + \boldsymbol{k}\frac{\partial}{\partial z}(x\dot{p}_x+y\dot{p}_y+z\dot{p}_z)\right]$$

$$\nabla[\frac{\hat{\boldsymbol{r}}\cdot\dot{\boldsymbol{p}}(t_0)}{r}] = \nabla[\frac{\boldsymbol{r}\cdot\dot{\boldsymbol{p}}(t_0)}{r^2}] \cong \frac{1}{r^2}\nabla(\boldsymbol{r}\cdot\dot{\boldsymbol{p}}(t_0))$$

$$= \frac{1}{r^2}\left[\boldsymbol{i}(\dot{p}_x + x\frac{\partial}{\partial x}\dot{p}_x + y\frac{\partial}{\partial x}\dot{p}_y + z\frac{\partial}{\partial x}\dot{p}_z) + \boldsymbol{j}(\dot{p}_y + x\frac{\partial}{\partial y}\dot{p}_x + y\frac{\partial}{\partial y}\dot{p}_y + z\frac{\partial}{\partial y}\dot{p}_z) + \boldsymbol{k}(\dot{p}_z + x\frac{\partial}{\partial z}\dot{p}_x + y\frac{\partial}{\partial z}\dot{p}_y + z\frac{\partial}{\partial z}\dot{p}_z)\right]$$

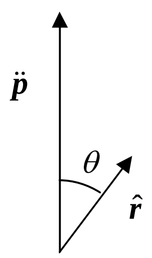
$\frac{\dot{p}_x}{r^2}, \frac{\dot{p}_x}{r^2}, \frac{\dot{p}_x}{r^2}$を無視

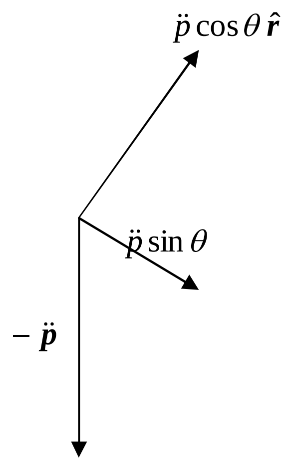
$$\cong \frac{1}{r^2}\left[\boldsymbol{i}(x\frac{\partial t_0}{\partial x}\frac{\partial\dot{p}_x}{\partial t_0} + y\frac{\partial t_0}{\partial x}\frac{\partial\dot{p}_y}{\partial t_0} + z\frac{\partial t_0}{\partial x}\frac{\partial\dot{p}_z}{\partial t_0}) + \boldsymbol{j}() + \boldsymbol{k}()\right]$$

$$= \frac{1}{r^2}\left[\boldsymbol{i}(x\frac{\partial\dot{p}_x}{\partial t_0} + y\frac{\partial\dot{p}_y}{\partial t_0} + z\frac{\partial\dot{p}_z}{\partial t_0})\frac{\partial t_0}{\partial x} + \boldsymbol{j}() + \boldsymbol{k}()\right]$$

$$= \frac{1}{r^2}(\boldsymbol{r}\cdot\ddot{\boldsymbol{p}})\nabla t_0$$

$\nabla\times\dot{\boldsymbol{p}}(t_0)$　x成分 $\frac{\partial\dot{p}_z}{\partial y} - \frac{\partial\dot{p}_y}{\partial z} = \frac{\partial t_0}{\partial y}\frac{\partial\dot{p}_z}{\partial t_0} - \frac{\partial t_0}{\partial z}\frac{\partial\dot{p}_y}{\partial t_0}$





$$\nabla \frac{Q(t-r/c)}{r} = \frac{1}{r}\nabla Q + Q\nabla\frac{1}{r}$$
$$= \frac{1}{r}\frac{\partial t_0}{\partial \boldsymbol{r}}\frac{dQ}{dt_0} - \frac{Q}{r^2}\hat{\boldsymbol{r}}$$
$$= -\frac{1}{cr}\frac{\boldsymbol{r}}{r}\dot{Q} - \frac{Q}{r^2}\hat{\boldsymbol{r}}$$
$$= -\frac{\dot{Q}}{cr}\hat{\boldsymbol{r}} - \frac{Q}{r^2}\hat{\boldsymbol{r}}$$

11.2 Point Charges

(11.66) $\rightarrow$ (11.68)

$$\boldsymbol{E}_{\text{rad}} = \frac{q}{4\pi\varepsilon_0}\frac{R}{(\boldsymbol{R}\cdot\boldsymbol{u})^3}[\boldsymbol{R}\times(\boldsymbol{u}\times\boldsymbol{a})] = \frac{q}{4\pi\varepsilon_0}\frac{R}{(cR)^3}[\boldsymbol{R}\times(c\hat{\boldsymbol{R}}\times\boldsymbol{a})] = \frac{q}{4\pi\varepsilon_0 c^2 R}[\hat{\boldsymbol{R}}\times(\hat{\boldsymbol{R}}\times\boldsymbol{a})] = \frac{q}{4\pi\varepsilon_0 c^2 R}[(\hat{\boldsymbol{R}}\cdot\boldsymbol{a})\hat{\boldsymbol{R}} - (\hat{\boldsymbol{R}}\cdot\hat{\boldsymbol{R}})\boldsymbol{a}]$$

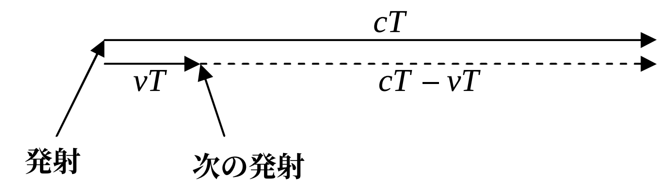
$\because\ \boldsymbol{u} = c\hat{\boldsymbol{R}} \quad \boldsymbol{R}\cdot\boldsymbol{u} = cR$

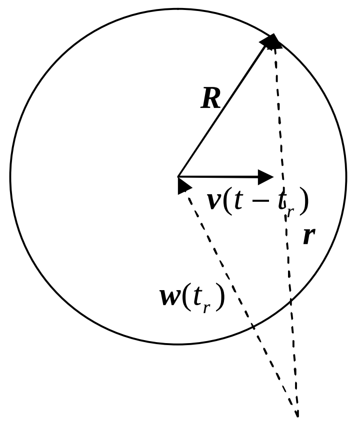
(11.68) $\rightarrow$ (11.69)

$$[(\hat{\boldsymbol{R}}\cdot\boldsymbol{a})\hat{\boldsymbol{R}} - \boldsymbol{a}]\cdot[(\hat{\boldsymbol{R}}\cdot\boldsymbol{a})\hat{\boldsymbol{R}} - \boldsymbol{a}] = (\hat{\boldsymbol{R}}\cdot\boldsymbol{a})^2 + a^2 - 2(\hat{\boldsymbol{R}}\cdot\boldsymbol{a})^2 = a^2 - (\hat{\boldsymbol{R}}\cdot\boldsymbol{a})^2$$

(11.70) $\int_0^\pi \sin^3\theta\, d\theta = \dfrac{4}{3}$

T : **発射周期** $\quad N_g = \dfrac{1}{T}$





$R = c(t - t_r)$

$\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{w}(t_r)$

$$N_t = \frac{1}{\frac{cT - vT}{c}} = \frac{c}{c-v}N_g$$

$$N_g = \frac{c-v}{c}N_t = (1-\frac{v}{c})N_t = (1-\frac{\hat{\boldsymbol{R}}\cdot\boldsymbol{v}}{c})N_t$$

(11.66) $\boldsymbol{E}_{\text{rad}} = \dfrac{q}{4\pi\varepsilon_0}\dfrac{R}{(\boldsymbol{R}\cdot\boldsymbol{u})^3}[\boldsymbol{R}\times(\boldsymbol{u}\times\boldsymbol{a})]$

(11.72)
$$\frac{dP}{d\Omega} = \left(\frac{\boldsymbol{R}\cdot\boldsymbol{u}}{Rc}\right)\frac{1}{\mu_0 c}E_{\text{rad}}^2 R^2 = \left(\frac{\boldsymbol{R}\cdot\boldsymbol{u}}{Rc}\right)\frac{1}{\mu_0 c}\left\{\frac{q}{4\pi\varepsilon_0}\frac{R}{(\boldsymbol{R}\cdot\boldsymbol{u})^3}[\boldsymbol{R}\times(\boldsymbol{u}\times\boldsymbol{a})]\right\}^2 R^2$$
$$= \left(\frac{\boldsymbol{R}\cdot\boldsymbol{u}}{Rc}\right)\frac{1}{\mu_0 c}\left(\frac{q}{4\pi\varepsilon_0}\right)^2\frac{R^4}{(\boldsymbol{R}\cdot\boldsymbol{u})^6}|\boldsymbol{R}\times(\boldsymbol{u}\times\boldsymbol{a})|^2$$
$$= \frac{1}{\mu_0 c^2}\left(\frac{q}{4\pi\varepsilon_0}\right)^2\frac{R^3}{(\boldsymbol{R}\cdot\boldsymbol{u})^5}|\boldsymbol{R}\times(\boldsymbol{u}\times\boldsymbol{a})|^2$$
$$= \varepsilon_0\left(\frac{q^2}{16\pi^2\varepsilon_0^{\,2}}\right)\frac{1}{(\hat{\boldsymbol{R}}\cdot\boldsymbol{u})^5}|\hat{\boldsymbol{R}}\times(\boldsymbol{u}\times\boldsymbol{a})|^2 = \frac{q^2}{16\pi^2\varepsilon_0}\frac{|\hat{\boldsymbol{R}}\times(\boldsymbol{u}\times\boldsymbol{a})|^2}{(\hat{\boldsymbol{R}}\cdot\boldsymbol{u})^5}$$