相対論

12.2.4

p.517

(12.64) $$\frac{d\boldsymbol{p}}{dt}\cdot\boldsymbol{u}=\frac{d}{dt}\left(\frac{m\boldsymbol{u}}{\sqrt{1-u^2/c^2}}\right)\cdot\boldsymbol{u}$$

$$=\frac{m}{\sqrt{1-u^2/c^2}}\frac{d\boldsymbol{u}}{dt}\cdot\boldsymbol{u}+\frac{d}{dt}\left(\frac{1}{\sqrt{1-u^2/c^2}}\right)m\boldsymbol{u}\cdot\boldsymbol{u}$$

$$=\frac{m}{\sqrt{1-u^2/c^2}}\frac{d\boldsymbol{u}}{dt}\cdot\boldsymbol{u}-\frac{1}{2}\frac{1}{(1-u^2/c^2)^{3/2}}\frac{d}{dt}\left(-u^2/c^2\right)mu^2$$

$$=\frac{m}{\sqrt{1-u^2/c^2}}\frac{d\boldsymbol{u}}{dt}\cdot\boldsymbol{u}+\frac{1}{2}\frac{1/c^2}{(1-u^2/c^2)^{3/2}}\frac{d}{dt}(\boldsymbol{u}\cdot\boldsymbol{u})mu^2$$

$$=\frac{m}{\sqrt{1-u^2/c^2}}\frac{d\boldsymbol{u}}{dt}\cdot\boldsymbol{u}+\frac{mu^2/c^2}{(1-u^2/c^2)^{3/2}}\boldsymbol{u}\cdot\frac{d\boldsymbol{u}}{dt}$$

$$=m\left[\frac{1}{(1-u^2/c^2)^{1/2}}+\frac{u^2/c^2}{(1-u^2/c^2)^{3/2}}\right]\boldsymbol{u}\cdot\frac{d\boldsymbol{u}}{dt}$$

$$=m\left[\frac{1-u^2/c^2+u^2/c^2}{(1-u^2/c^2)^{3/2}}\right]\boldsymbol{u}\cdot\frac{d\boldsymbol{u}}{dt}=\frac{m\boldsymbol{u}}{(1-u^2/c^2)^{3/2}}\cdot\frac{d\boldsymbol{u}}{dt}$$

$$\frac{d}{dt}\left(\frac{mc^2}{\sqrt{1-u^2/c^2}}\right)=mc^2\left[-\frac{1}{2}\frac{1}{(1-u^2/c^2)^{3/2}}\frac{d}{dt}\left(-\frac{\boldsymbol{u}\cdot\boldsymbol{u}}{c^2}\right)\right]=\frac{m\boldsymbol{u}}{(1-u^2/c^2)^{3/2}}\cdot\frac{d\boldsymbol{u}}{dt}$$

p.527　Ex. 12.13

$$\gamma_0=1/\sqrt{1-v_0^2/c^2}\quad R_x=R\cos\theta\quad\sqrt{R_y^2+R_z^2}=R\sin\theta$$

$$E_x=E_{x0}=\frac{1}{4\pi\varepsilon_0}\frac{qx_0}{(x_0^2+y_0^2+z_0^2)^{3/2}}=\frac{1}{4\pi\varepsilon_0}\frac{q\gamma_0R_x}{(\gamma_0^2R_x^2+R_y^2+R_z^2)^{3/2}}$$

$$E_y=\gamma_0E_{y0}=\frac{1}{4\pi\varepsilon_0}\frac{\gamma_0qy_0}{(x_0^2+y_0^2+z_0^2)^{3/2}}=\frac{1}{4\pi\varepsilon_0}\frac{\gamma_0qR_x}{(\gamma_0^2R_x^2+R_y^2+R_z^2)^{3/2}}$$

$$E_z=\gamma_0E_{z0}=\frac{1}{4\pi\varepsilon_0}\frac{\gamma_0qz_0}{(x_0^2+y_0^2+z_0^2)^{3/2}}=\frac{1}{4\pi\varepsilon_0}\frac{\gamma_0qR_z}{(\gamma_0^2R_x^2+R_y^2+R_z^2)^{3/2}}$$

$$\boldsymbol{E}=\frac{1}{4\pi\varepsilon_0}\frac{\gamma_0q\boldsymbol{R}}{(\gamma_0^2R^2\cos^2\theta+R^2\sin^2\theta)^{3/2}}=\frac{1}{4\pi\varepsilon_0}\frac{\dfrac{q}{\sqrt{1-v_0^2/c^2}}}{\left(\dfrac{R^2\cos^2\theta}{1-v_0^2/c^2}+R^2\sin^2\theta\right)^{3/2}}\boldsymbol{R}$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{(1-v_0^2/c^2)^{3/2}\dfrac{q}{\sqrt{1-v_0^2/c^2}}}{[R^2\cos^2\theta+(1-v_0^2/c^2)R^2\sin^2\theta]^{3/2}}\boldsymbol{R}=\frac{1}{4\pi\varepsilon_0}\frac{(1-v_0^2/c^2)q}{[1-(v_0^2/c^2)\sin^2\theta]^{3/2}}\frac{\boldsymbol{R}}{R^3}$$