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Comprehensive Gröbner systems in CoCoA

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1 Comprehensive Gröbner systems

The concepts of a comprehensive Gröbner system (CGS) was introduced by Weispfenning [6] to associate Gröbner basis like objects for parametric polynomial systems. For a specialization of parameters, a Gröbner basis of the specialized ideal can be immediately recovered from a branch of the associated CGS. This property of CGS make them attractive in applications where a family of related problems can be parameterized and specified using a parametric polynomial system.

Several improvements have been done by Weispfenning (CCGB [7]), Montes (DISPGB [3]) and Suzuki-Sato (ACGB [4]). However, all these algorithms essentially require computations in polynomial ring over a coefficient field of rational functions, $K(A)[X]$, where K is the ground field and A are the parameters and X the actual indeterminates, together with delicate handling the case distinctions over the parameters. This last fact makes these algorithms hard to implement in computer algebra systems.

In 2006 Suzuki-Sato [5] introduced a new approach to compute comprehensive Gröbner systems and comprehensive Gröbner bases. Making good use of some results by Kalkbrener [2], their algorithms do not require case distinctions to be pairwise disjoint and just need the final result of Gröbner bases in $K[A, X]$, so that that can be easily implemented in any computer algebra system. Suzuki and Sato implemented their algorithms in several computer algebra systems such as Risa/Asir, Singular and Maple and proved that it is sufficiently fast comparing with existing implementations when there are few parameters.

This approach attracted our attention and we decided to implement their algorithm also in the computer algebra system CoCoA [1]. We designed some optimization and made comparisons, and we wish to present and discuss our work in progress.

Keywords
Comprehensive Gröbner systems, Gröbner bases, CoCoA

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