Applications of Computer Algebra – ACA 2021
Virtual. Online | July 23-27, 2021
Session on *"Algorithms for Polynomial System Solving and their Applications"*

An Attempt to Enhance Buchberger's Algorithm by Using PRSs and GCDs (a Brief Survey)

Tateaki Sasaki[1]***, Masaru Sanuki***[2]***, Daiju Inaba***[3]***, Fujio Kako***[4] [sasaki@math.tsukuba.ac.jp]

[1] Professor emeritus, University of Tsukuba, Tsukuba-shi, Ibaraki 305-8571, Japan
[2] Faculty of Medicine, University of Tsukuba, Tsukuba-shi, Ibaraki 305-8571, Japan
[3] The Mathematics Certification Institute of Japan, Ueno 5-1-1, Tokyo 110-0005, Japan
[4] Nara-Women's University (previous affiliation), Nara-shi, Nara 630-8506, Japan

By GB we denote the reduced Gröbner basis of polynomial ideal w.r.t. the lexicographic term order. Let the GB of given three-or-more polynomial system $\mathcal{F}$ be $\mathrm{GB}(\mathcal{F}) = \{\widehat{G}_1, \widehat{G}_2, \cdots\}$, where $\widehat{G}_1 \prec \widehat{G}_2 \prec \cdots$. This talk surveys our recent works for computing small multiples or leading-monomial multiples (multiplier is 1 sometimes) of important elements of $\mathrm{GB}(\mathcal{F})$, by the PRSs and GCDs. Let the multiples be $\widetilde{G}_1 \prec \widetilde{G}_2 \prec \cdots$. Our plan is to compute $\mathrm{GB}(\mathcal{F})$ by applying Buchberger's algorithm to $\mathcal{F} \cup \{\widetilde{G}_1, \widetilde{G}_2, \cdots\}$. Our method is unique in that the multiples are computed as $\widetilde{G}_1 \Rightarrow \widetilde{G}_2 \Rightarrow \cdots$. We note that the coefficient sizes of actual elements of GB are such that $\mathrm{csize}(\widehat{G}_1)$ is almost the smallest among $\mathrm{csize}(\widehat{G}_1), \mathrm{csize}(\widehat{G}_2), \cdots$. This fact suggests us that our approach is reasonable.

Two new theorems are proved, one is for computing the lowest-order element of ideal generated by relatively prime $G, H \in \mathbb{Q}[x, u_1, u_2, \ldots]$, and another is for computing small multiples of elements of $\mathrm{GB}(\mathcal{F})$ efficiently. Two propositions are given for removing still remaining extraneous factors effectively. Four new concepts are introduced, "healthy system", "rectangular PRSs", "elimination of LC (leading coefficient) set", and "LCtoW (LC to Whole) polynomial". We explain these by using many examples.

Keywords
lexicographic Gröbner basis, polynomial remainder sequence, coefficients of generators

References

[1] T. SASAKI; D. INABA, Simple relation between the lowest-order element of ideal $\langle G, H\rangle$ and the last element of the polynomial remainder sequence. In *Proceedings of SYNASC 2017*, Tudor Jebelean et al. (eds.), 55–62 (2018).

[2] T. SASAKI; D. INABA, Computing the lowest-order element of the elimination ideal of multivariate polynomial system by using remainder sequences. In *Poceedings of SYNASC 2018*, Erika Abraham et al. (eds.), 37–44 (2019).

[3] T. SASAKI, An attempt to emhance Buchberger's algorithms by using remainder sequences and GCD operation. In *Proccedings of SYNASC 2019*, 27–34 (2020).

[4] T. SASAKI; M. SANUKI; D. INABA; F. KAKO, An attempt to enhance Buchberger's algorithm by using remainder sequences and GCDs (II). *RIMS Kôkyûroku (Research Reports of Research-Inst.-for-Mathematical-Sciences, Kyoto Univ.) 2185*, 71–80 (2021).