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Proposal of Multivariate Polynomial Arithmetic in a Specified Width of High- or Low-Exponents

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The truncated power-series is very useful in computer algebra, however we must control the cutoff degree very carefully when we use the power-series in actual algorithms. In this paper, we propose a much more useful multivariate polynomial arithmetic on the recursive representation of the polynomial. The arithmetic reserves only the terms of high- or low-exponents in a user-specified width w.r.t. a specified variable (other terms are discarded automatically). Of course, the exponents are determined by the conventional arithmetic hence changed as the computation proceeds, while the width (measured from either the highest or the lowest exponent) is fixed.

We obtained this idea in our recent project of enhancing Buchberger's algorithm for the lexicographic Gröbner bases of multivariate polynomial ideals, by using PRSs (Polynomial Remainder Sequences) and GCDs. In our project, PRSs and their "coefficients of generators" (= coefficients of Bezout's identity) are critically important but the computation of them is very heavy. However, what we need are only low-exponents parts in a narrow width, say 1/10 of the exponent range of the full expression. We investigate this idea from practical point of view, and give detailed procedures for realizing this idea definitely. We show several examples of using new arithmetic.

Keywords
polynomial remainder sequence, extended Euclidean algorithm, coefficients of generators

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