Some tips on the implementation of CGS in SageMath

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A comprehensive Gröbner system (CGS) is a powerful tool for handling parametric polynomial systems. Its first practical computation algorithm was introduced in [6]. With improvements of the subsequent works such as [3,4,5], we now have several its application programs such as the one introduced in [2].

It seems that a basic framework of its practical computation algorithm was established by the work of [5] at least from a theoretical point of view, however, there still remain many important issues concerning its efficient implementation. We have developed several techniques which improve the existing implementations of CGS. Our methods require several ideal manipulations such as the computations of radical ideals or saturation ideals. Even though those computations are rather heavy in general, we have observed our techniques are quite effective through our implementation in SageMath [1].

In the talk, we introduce our techniques through an interesting example of geometry theorem proving(discovery) which is one of the most typical applications of CGS.

Keywords

Comprehensive Gröbner System, SageMath

References

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